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ACI
PROCEEDINGS
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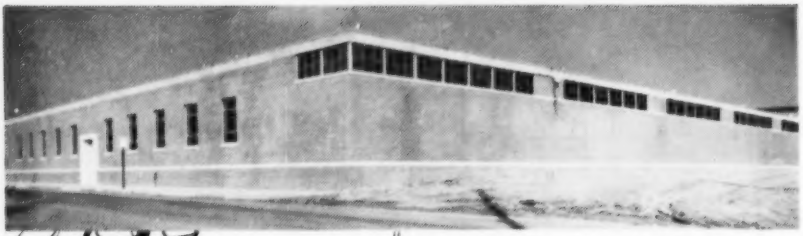
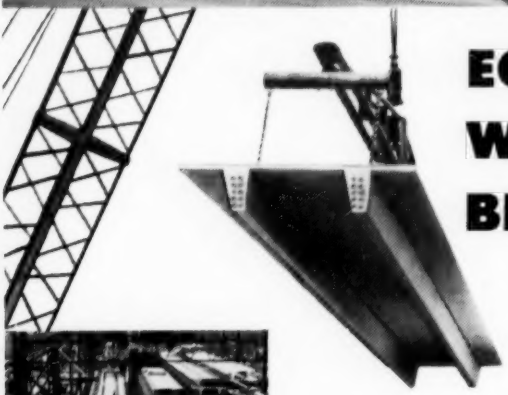
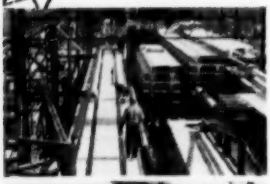




ECONOMY WITH A BIG E



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The ACI JOURNAL technical pages are a medium for reporting what is of current interest in the concrete field; in research, design, construction, maintenance, or manufacturing. Members and readers are urged to "have their say" as a means of sharing knowledge and experience; time and changing interests demand that new names and new ideas take their places beside the old. Contributions describing new materials and techniques or new applications of familiar practices are welcomed by the Institute.

To guide the flow of material the Board of Direction has assigned to the Technical Activities Committee the selection of papers, committee reports, discussions, and other contributions for JOURNAL publication. To assist in their technical appraisal a corps of experts in various segments of the field volunteer their services. From the technical standpoint, acceptable contributions present original material for improvement of design, manufacturing, or construction; confirm, revise, or upset established ideas or practices; or review, digest, or arrange accumulated experience for more ready use.

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In ACI Journal next month

"Design Considerations for Concrete Pavement Reinforcement for Crack Control," is the report of SUBCOMMITTEE IV, ACI COMMITTEE 325, which devotes attention to the characteristics of subgrade frictional resistance, and to the relation between steel stress and crack width. A rational method for determining the coefficient of subgrade resistance is proposed.

ERIC C. MOLKE discusses design and construction features of the Springfield, Mo., "Auditorium Framed with Prestressed Roof Girders." The 146-ft girders were cast of 500-psi concrete and prestressed on the ground; then they were raised between twin columns by the same jacks used for the prestressing. Stability requirements during construction and for the completed structure are considered.

Introductory paper of a four-part series on lightweight concrete, "Properties of Lightweight Aggregates and Lightweight Concretes," is presented by G. W. WASHA as a part of the work of ACI Committee 213. Lightweight concrete is divided into: cellular or foam concrete; no-fines or "popcorn" concrete; and lightweight-aggregate concrete. Properties of both concrete and aggregates are summarized.

"Structural Lightweight-Aggregate Concrete" is the second part of the series. RALPH W. KLUGE has attempted to gather all of the related data available, both published and unpublished, and to present this information as a guide in the use of lightweight aggregates for structural concrete. He discusses proportioning and control of concrete mixtures containing lightweight aggregate, and summarizes structural properties of lightweight-aggregate concrete.

Analysis of a helicoidal staircase involves torsional moments as well as bending moments and shears, says VICTOR R. BERGMAN in his paper, "Helicoidal Staircases of Reinforced Concrete." The method used in this paper reduces the staircase to its horizontal projection, and the problem becomes essentially that of a fixed-end, curved beam loaded normal to its plane of curvature.

A. M. OZELL and E. ARDAMAN are the authors of "Fatigue Tests of Pre-Tensioned Prestressed Beams." Results of their pilot tests indicate that fatigue failures were caused by breaking of wires in the prestressing strand, and not as a result of bond failure. Their tests also indicated a flexural fatigue strength approximately 1.8 times the design load.

Title No. 53-13

Strength of a Concrete Slab Prestressed in Two Directions*

By A. C. SCORDELIS,† K. S. PISTER,‡ and T. Y. LIN§

SYNOPSIS

Elastic behavior and ultimate strength of a full size concrete slab prestressed in two directions were investigated. Prestressing was accomplished by means of unbonded post-tensioned cables. The slab was supported only at the four corners, simulating a lift flat slab. It was loaded uniformly by means of air pressure in plastic bags. Experimental deflections and strains were checked against the classical elastic theory. Observed ultimate strength was compared to that obtained by the crack-line theory as applied to prestressed slabs.

INTRODUCTION

With the advent of lift-slab construction the question of the behavior of concrete flat slabs prestressed in two directions has become one of prime importance. A number of structures using the lift-slab method of construction and prestressed concrete slabs have been built in this country, but as yet few basic data are available regarding the behavior of such slabs. The investigation reported herein considers the fundamental case of a square slab simply supported at the four corners and uniformly loaded (Fig. 1).

The purpose of the investigation was to determine the behavior, through and above the elastic range, of a concrete slab prestressed in two directions. Some of the questions which the investigation endeavored to answer for this type of slab were: (1) Is the elastic theory valid up to occurrence of cracks? (2) Can the cracking load be predicted by elastic theory, using the flexural tensile strength as determined by plain concrete specimens? (3) What is the physical behavior of such a slab through the plastic range and finally under ultimate load? Does it deflect excessively? Is the failure sudden or gradual? (4) What is the comparison between the actual ultimate strength of such a slab and that given by the crack-line theory and the ultimate strength of unbonded prestressed beams?

The prediction of the cracking load in slabs prestressed in two directions has

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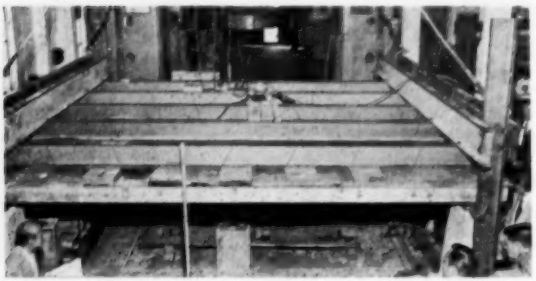

Fig. 1—General test arrangement

received the attention of various investigators. Some engineers believe that there exists sufficient plastic behavior of concrete in tension prior to cracking so as to delay appreciably the appearance of cracks.* Others maintain that the elastic theory may be used to compute the cracking load within experimental limits.† As a further attempt to check the validity of the elastic theory, a full size slab was tested under laboratory-controlled conditions to insure accuracy. The slab was loaded to failure, thus yielding data for its behavior at ultimate load.

DESCRIPTION OF TEST SLAB

The test slab shown in Fig. 2 had over-all dimensions of 15 x 15 ft in plan and was 5 in. thick. Supports at the four corners were on 14-ft centers in both directions. A steel plate with a bearing area of 4 x 8 in. was used at each support. A rocker and roller arrangement as detailed in Fig. 3 permitted the necessary rotations and horizontal movement at the supports so that no restraint was introduced at these points.

The slab was post-tensioned with ten cables in each direction. All cables were laid straight with no drape. Each cable consisted of six wires of ¼-in. diameter with cold-formed buttons at the ends of the wires. The wires were greased and wrapped with paper and were anchored to the concrete with Prescon anchorages. Over-all diameter of the cables was ¾-in. It was desired to approximate a condition of uniform prestress in both directions. In an east-west direction, ten cables spaced 1 ft 6 in. center to center were placed at middepth of the slab. In a north-south direction, ten cables spaced also 1 ft 6 in. center to center were alternated one just above and then one just below the east-west cables; the centers of these cables were therefore 1¾ in. from the top and bottom surfaces of the slab, respectively. To preserve symmetry this resulted in six cables below and four cables above the east-west cables, as shown in Fig. 2.

In addition to the prestressing steel, a small amount of intermediate-grade reinforcement was placed at each corner of the slab. This consisted of four #3 bars, 3 ft long, at 9-in. centers running both ways at each corner. This steel was included

*Freyssinet, E., "Importance et Difficultés de la Mecanique des Betons." *Proceedings*, First International Congress on Prestressed Concrete, Ghent, Belgium, 1951. (English translation in *Magazine of Concrete Research*, [London] No. 8, 1951.)

†Rogers, G. L., "Validity of Certain Assumptions in the Mechanics of Prestressed Concrete," ACI JOURNAL, Dec. 1953, *Proc.* V. 50, pp. 317-322.

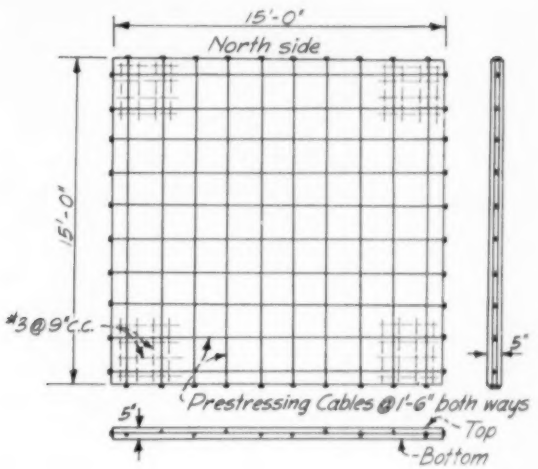

Fig. 2—Plan and sections of test slab showing steel arrangement

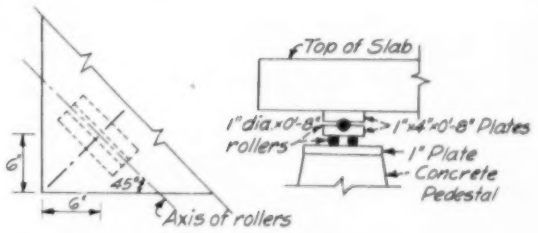

Fig. 3—Detail of plan and elevation at support

to minimize the possibility of local failure at the supports and also to distribute the load during lifting of the slab. The slab was cast on the floor and lifted to position with a 10-ton overhead crane. Pick-up points were provided at the four corners of the slab.

MATERIALS

Concrete for the slab was designed to possess a minimum strength of 5000 psi at 28 days using 6 x 12-in. cylinders. The concrete was delivered ready-mixed from a central batching plant. The mix contained 7 sacks of Type I cement per cu yd of concrete. The water-cement ratio was 5.3 gal. per sack. The aggregate consisted of Elliot top sand and gravel having a maximum size of ¾ in. Batch proportions by weight, based on saturated surface dry conditions, were water, 0.47 : cement, 1.00 : sand, 2.05 : gravel, 2.58. The slump was 2 in. at the start, decreasing to about ½ in. near the completion of placing which lasted about 1½ hr.

The slab was cured with a moist burlap covering for 8 days and then left in air-dry condition. All control specimens were cured in a similar manner to the slab.

Control specimens consisted of twelve 6 x 12-in. cylinders and six 4 x 6 x 20-in. concrete beams. Three cylinders were tested in compression at 7, 11, 29, and 50 days.

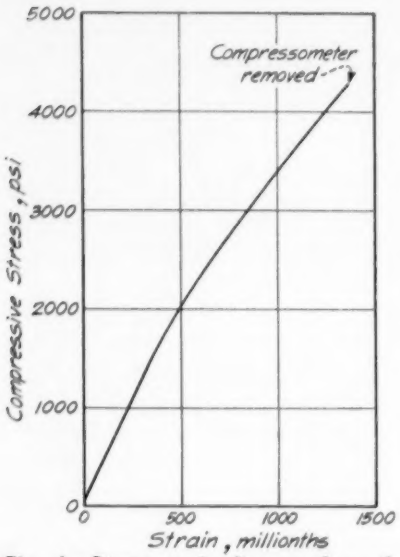

Fig. 4—Stress-strain diagram for a 6 x 12-in. cylinder at an age of 50 days

Average values for the compressive strength and modulus of elasticity of these cylinders are given in Table 1. A typical stress-strain diagram is shown in Fig. 4. Three 4 x 6 x 20-in. beams were tested, under third-point loading on an 18-in. span, at 11 and 50 days. Average values for the moduli of rupture as obtained for these beams are also listed in Table 1.

The prestressing cables each contained six ¼-in. diameter cold drawn steel wires. The wire possessed an ultimate strength of 250,000 psi with a proportional limit of 155,000 psi and a yield point of 214,000 psi measured by 0.2 percent offset method. Total elongation was 5.6 percent in a gage length of 10 in. The modulus of elasticity was 28.4 $\times 10^6$ psi up to the proportional limit. A typical stress strain diagram is shown in Fig. 5.

MANUFACTURE

The slab was cast on the floor, using a plywood soffit and 2 x 5-in. side forms. The prestressing cables and mild steel were first placed in position and the concrete then placed. Prestressing was performed at an age of 11 days. The sequence was to first prestress the center cables in each direction and then work progressively outward to the cables along the edges of the slab.

The prestressing steel was initially stressed to 150,000 psi. Loss of prestress due to creep and shrinkage in concrete and creep in steel was not measured because from previous experience in the laboratory, for the same kind of steel

TABLE 1—PROPERTIES OF CONCRETE

Age, days	7	11	29	50
Compressive strength of 6 x 12-in. cylinders, psi	4110	4930	5790	5820
Average tangent modulus of elasticity between 400 and 1400 psi of 6 x 12-in. cylinders, psi	—	3.73×10^6	4.15×10^6	4.19×10^6
Modulus of rupture of 4 x 6 x 20-in. beams on 18-in. span under third-point loading, psi	—	463	—	598

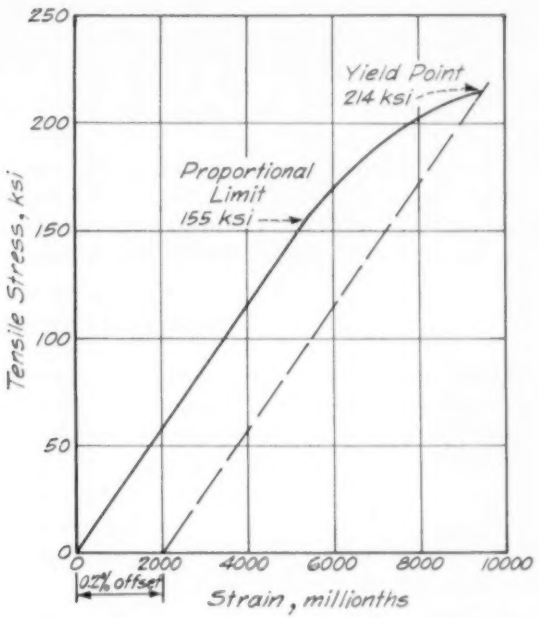

Fig. 5—Stress-strain diagram for prestressing wires

and concrete, a loss of 15 percent was known to be a close estimate. Effective prestress in the steel was therefore taken as 128,000 psi.

Prior to lifting the slab from the floor, elevations were determined at each of the four support locations by an engineer's level. The slab was then lifted from the floor and placed on the four supports. Levels were then taken again and adjustments made with shims so that the four support points had the same relative elevations as those prior to lifting. This procedure was followed to insure that the reactions at each of the supports would be equal and was necessitated by the external indeterminacy of the structure.

METHOD OF LOADING

It was desired to have a method of loading the slab which would satisfy the following requirements: (1) uniform load over the entire slab, (2) speed in applying and removing the load, (3) relatively accurate measurements of each load increment, and (4) low labor cost.

A plan was developed by which the slab was to be loaded with air pressure acting against a steel frame. The loading arrangement is shown in Fig. 6. Four plastic air bags covered the surface of the slab. Air pressure from the compressed air system of the laboratory was introduced at A, at 80 psi, and reduced at B to 10 psi. The control valve at C regulated the amount of air delivered to

the manifold at D. From the manifold, four lines, one to each bag, brought the air for the inflation of the bags. Air pressure was measured by a water manometer at E. Load could be removed by bleeding the air at F. With the arrangement described, the load on the slab could be controlled and measured to the nearest 2 or 3 psf.

The method of loading described above proved to be highly successful and appears to be an excellent method for obtaining distributed loads on slabs. Strip loading, loading over small areas, or uniform loading over the entire slab can be readily accomplished by this technique.

INSTRUMENTATION

Strains at various locations on the top and bottom surfaces of the slab were measured by SR4, type A9, strain gages, which have a gage length of 6 in. Twenty-four of these gages were employed, 16 being placed on the bottom and eight on the top surface of the slab. The location of these gages is indicated in Fig. 7. Because of the symmetry of the slab, its loading, and the gage locations, it was possible to obtain, in some cases, readings at two or more gage locations which could be averaged to give a more representative value for strain at a particular point in the slab.

To obtain deflection profiles along a transverse centerline as well as along an edge, deflection gages were placed as shown in Fig. 7. These were simple dial gages bearing on the bottom surface of the slab. The dials had a least count of 0.001 in. Dial gages were used until the load approached the expected ultimate value, after which they were removed. Deflections were then obtained by level readings taken on graduated scales hung from the bottom of the slab.

TEST PROGRAM

The test program consisted of four separate loading cycles. In each test cycle the load was brought up to a certain level and then completely removed. Strains and deflections were measured at load increments corresponding to 2 or 4 in. of water pressure (*i.e.*, 10.4 or 20.8 psf).

Test 1 was designed to record the behavior of the slab before cracking; test 2 to check the results of test 1 and to bring the load up to cracking; test 3 to determine the behavior of the slab, after it had been cracked, from zero load through the post-cracking range; and test 4 to check the results of test 3 and to bring the load up to the ultimate.

The range of test 1 at 46 days age was from 0 to 12 in. of water pressure (0–62 psf). Test 2 raised the maximum to 24 in. of water (125 psf) at 47 days; test 3, to 30 in. of water (156 psf) at 48 days; and test 4, to 46 in. of water (238 psf) at 50 days. Strain readings indicated cracking at 83 psf load, and cracks were visible at 104 psf load in test 2, 3, and 4. Failure occurred at 238 psf in test 4.

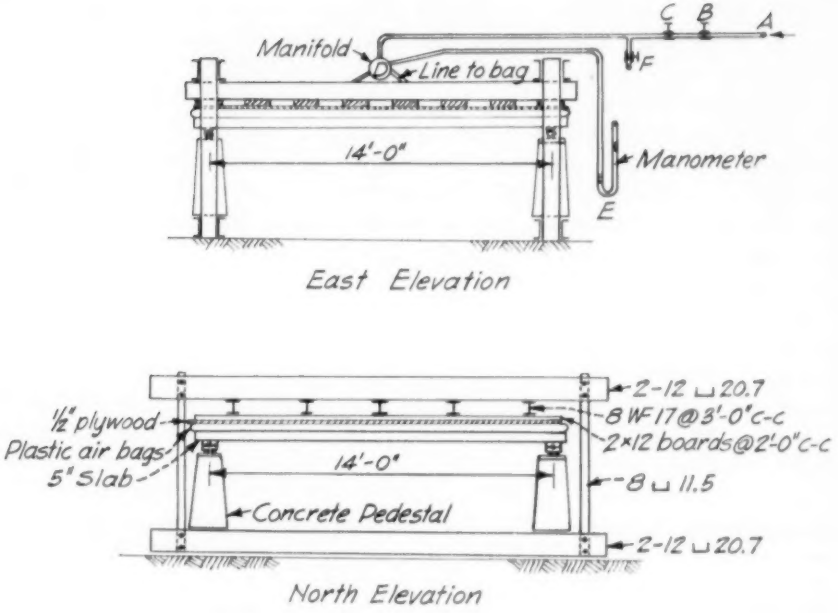

Fig. 6—Loading frame and air system

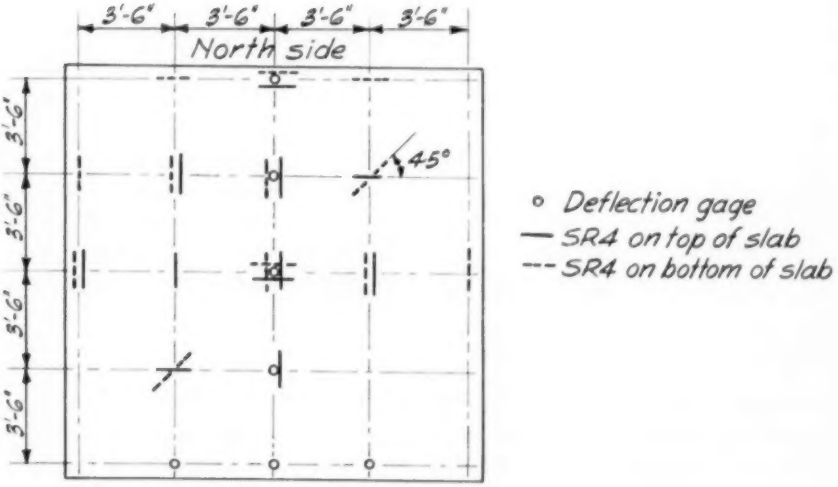

Fig. 7—Location of gages

Elastic theory

One of the main objectives of this investigation was to compare the experimental results with theoretical analysis. The analytical determination of deflections and bending moments in an elastic slab depends upon the possibility of solving the Lagrangian differential equation of slab theory, subject to the appropriate boundary conditions imposed by its loading and supports. The problem of a uniformly loaded slab supported by concentrated corner forces has been solved by Marcus.* The significant bending moments and deflections have been tabulated and these values form the basis for the comparison of test results and theory.

The use of this solution in the present problem is only an approximation, in view of the slight difference in method of support. In the test slab a 6-in. overhang extended on all sides. The effect of this overhang is neglected in applying the theory, and a 14-ft simple span is assumed. In addition, the results given by Marcus are based on a value of Poisson's ratio of 0.3. This value is undoubtedly too high for concrete; however, because of the manner in which Poisson's ratio occurs in the boundary conditions for a slab with a free edge, the effect of a change in Poisson's ratio on the solution of the problem can be ascertained only with considerable mathematical effort. It was felt that the effect would be less than the probable experimental errors. In computing the slab stiffness and reducing strain data to compute bending moments, however, a value of 0.2 was used for Poisson's ratio. Fig. 8 shows a quarter of the slab with the control points A, B, C, D, E, S, at which comparisons of test results with theory could be made.

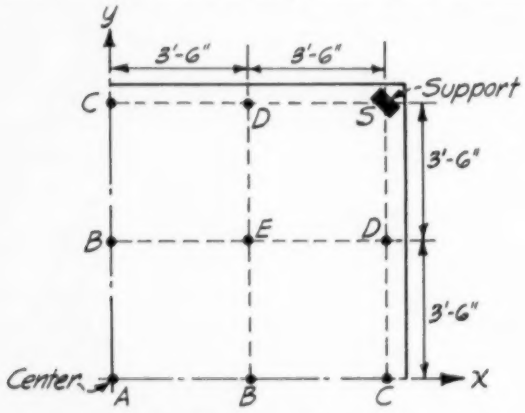

Fig. 8—Quarter of slab showing control points for comparison of test results with theory

Deflections, moments, and strains—comparison with elastic theory

Only typical results are presented from the series of tests. All deflections and strains referred to in the following discussion are those due to live loads;

*Marcus, H., *Die Theorie elastischer Gewebe und ihre Andwendung auf die Berechnung biegsamer Platten*, Julius Springer, Berlin, 1932, pp. 173-184.

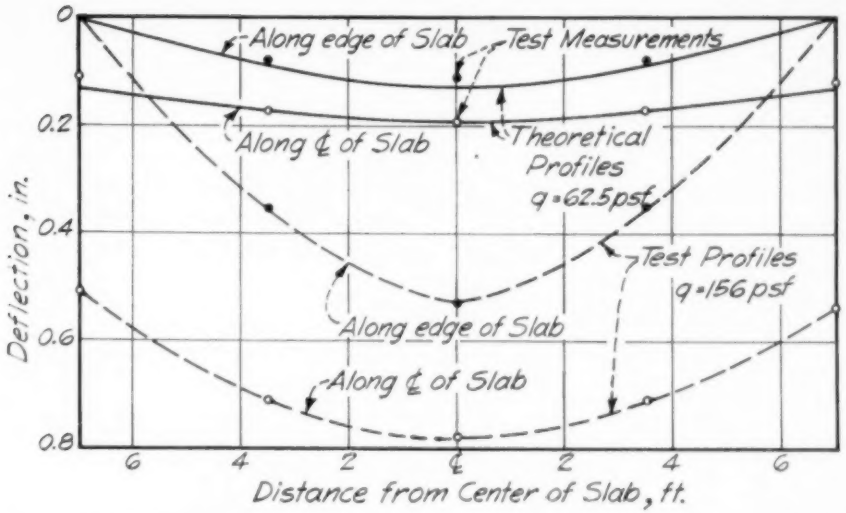

Fig. 9—Theoretical and test deflection profiles for q = 62.5 psf (test 2) and test deflection profiles for q = 156 psf (test 4)

the effect of dead load was not measured in these tests. First the theoretical deflection profiles along the centerline (ABC) and along the edge (CDS) as given by the Marcus theory are plotted on Fig. 9, corresponding to a load of 62.5 psf and a modulus of elasticity of 4,190,000 psi. On the same figure are plotted the measured deflections under the uniform load of 62.5 psf (test 2). Good agreement is seen to exist between measured and theoretical values within this elastic range since no cracking had yet occurred anywhere in the slab. The lower set of curves on Fig. 9 was plotted through the measured deflections for a load of 156 psf (test 4). Since the slab had been fairly well cracked at that load, the deflections were higher than those given by the elastic theory based on an uncracked section.

To study further the load-deflection relationship for center point A and edge point C, measured values for test 2 are plotted in Fig. 10. Two straight lines, representing the elastic theory for E_c = 4,190,000 psi, are also shown. Close agreement is again observed for loads up to about 80 psf, beyond which a faster increase in deflection indicated the beginning of some cracking in the slab under relatively high loads.

To record deflections of the slab during unloading, measurements were made at the end of test 3 as the load was gradually removed. Center deflections at point A are plotted in Fig. 11, giving the load-deflection relationship for a complete cycle of loading and unloading, with the maximum load well beyond the load at which cracking occurred. Note the divergence of the two curves, which can be partly explained by the residual deflection produced by creep and plasticity of concrete.

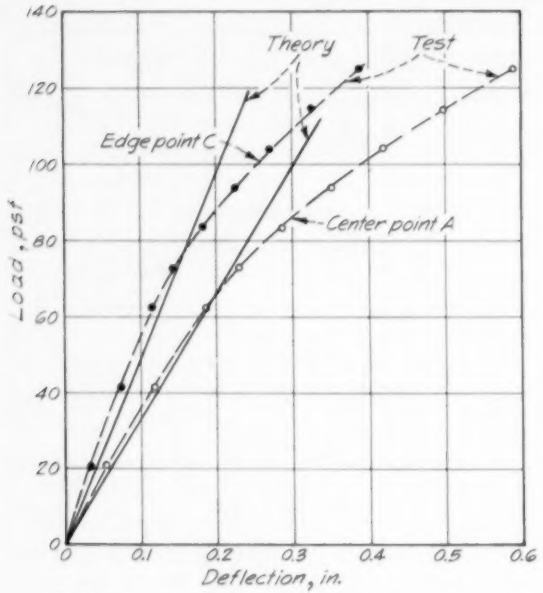

Fig. 10—Load-deflection curves (test 2)

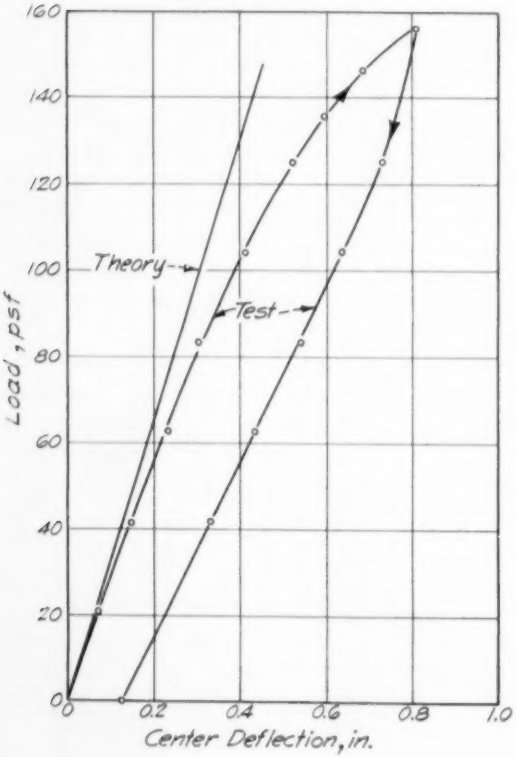

Fig. 11—Load-center deflection curve for one cycle of loading and unloading (test 3)

Center deflections for test 4 are plotted in Fig. 12. This gives the load-deflection relationship for the center of the slab over the entire range of loading up to the ultimate capacity. In comparing the initial slopes of the load-deflection curves for tests 2, 3, and 4 (Fig. 10, 11, and 12), it is seen that the stiffness of the slab decreased as cracking increased.

To compare the actual distribution of moments across the centerline of the slab with the theoretical values, curves for both M_x and M_y are plotted in Fig. 13 for a load of 62.5 psf. M_x and M_y are moments in the direction of the X and Y axes respectively. Rather close agreements are observed between the shape of the theoretical and experimental curves.

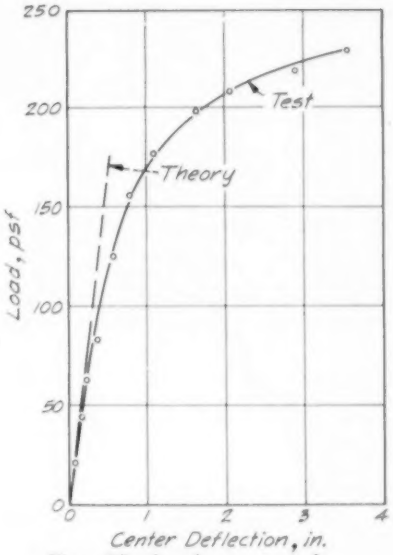

Fig. 12—Load – center deflection curve (test 4)

Experimental curves were based on a value of $E_c = 4{,}190{,}000$ psi as given by the cylinders. The theoretical and experimental curves would practically coin-

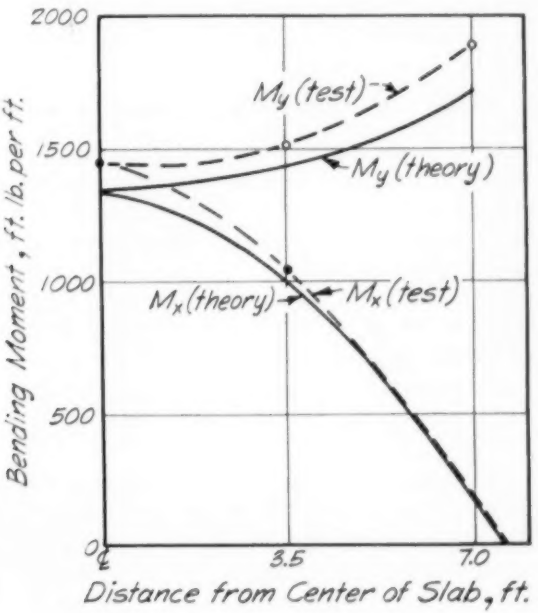

Fig. 13—Experimental and theoretical bending moments along slab centerline, q = 62.5 psf (test 1)

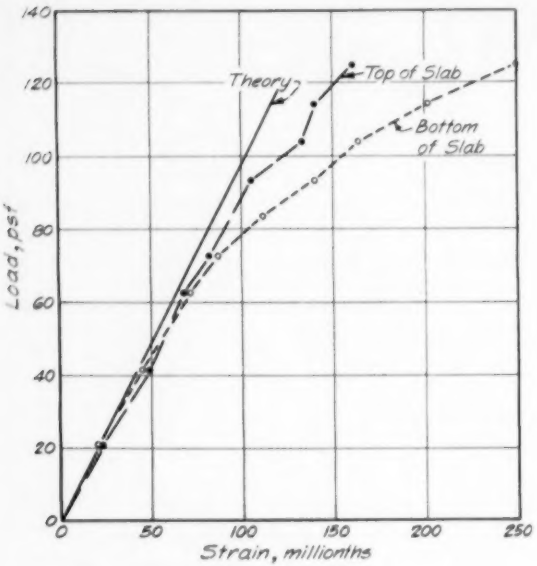

Fig. 14—Load versus strain at center of slab (test 2)

cide if E_c was assumed to be 3,900,000 psi.

Typical strain readings for the center of the slab are shown in Fig. 14, compared with the load-strain curve given by the elastic theory, again for E_c = 4,190,000 psi. The agreement is good below a load of about 80 psf, above which cracking of the slab resulted in higher strains in the bottom fibers, while the upward shifting of the neutral axis also indicated increased strains in the top fibers.

Cracking load—experimental versus theoretical

To compute the load producing the first cracks, the resisting moment of the slab at a tensile stress of 598 psi (the average modulus of rupture) is calculated as follows. For an effective concentric prestress of 0.294 × 128,000 = 37,600 lb per cable, the uniform compression in the concrete slab for cable spacing of 18 in. is:

$$f = \frac{F}{A_c} = \frac{37{,}600}{5 \times 18} = 418 \text{ psi}$$

Hence the resisting moment per ft of slab at a modulus of rupture of 598 psi is:

$$M = fI/c = fbd^2/6$$
$$= \frac{(418 + 598)\ 12 \times 5^2}{6 \times 12} = 4230 \text{ ft-lb}$$

As seen in Fig. 13, the maximum bending moment, hence the maximum fiber stress, occurs at the edge of the slab. The magnitude of this moment (from Marcus) is $M_y = 0.562\ qa^2$, where q = load in psf and a = ½ span. Equating this to the resisting moment of 4230 ft-lb at cracking, gives $0.562\ qa^2 = 4230$ for $a = 7$ ft, $q = 154$ psf. Since the dead load of the slab is 63 psf, cracking should

occur at a live load of 154 − 63 = 91 psf. The first crack was observed, at point C, at a load of 104 psf, while, as indicated by the strain and deflection curves of Fig. 10 and 14, minute cracks might have started at about 80 psf although they could not be observed visually even with the help of magnifying glasses.

Ultimate load—theoretical versus experimental

According to the crack-line theory of slab failure, the slab's ultimate capacity is not reached until a crack pattern has formed, making the slab unstable. This necessitates that cracks spread to the edges of the slab. A number of crack patterns are possible; the controlling one is that which will form under the smallest possible load. It was found for the test slab that this condition should occur

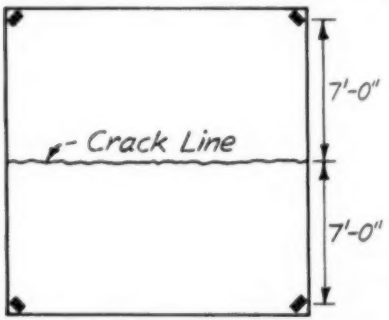

Fig. 15—Theoretical cracking pattern at ultimate load

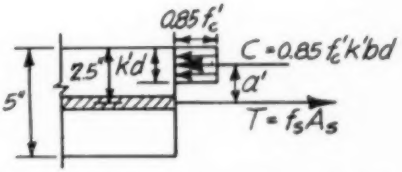

Fig. 16—Whitney theory for ultimate moment

for a crack pattern as shown in Fig. 15. Assuming that the prestressing steel has sufficient ductility the slab will reach its ultimate capacity when the moment across the entire crack has reached a uniform value equal to the ultimate resisting moment of the section.

The average external moment per ft of width of slab along the centerline crack is given by:

$$M = \frac{q(14)^2}{8} = 24.5q$$

The ultimate resisting moment per ft of width can be approximated by applying Whitney's theory using a rectangular stress block in the concrete as shown in Fig. 16.

$$M' = Ta' = Ca'$$
$$T = f_s A_s = C = 0.85\, f_c' k' bd$$

For $b = 12$ in.; $A_s = 12/18 \times$ area of one cable $= 12/18\ (0.294) = 0.196$ sq in.; $f_c' = 5820$ psi

$$k'd = \frac{f_s A_s}{0.85 f_c' b} = \frac{f_s\ (0.196)}{0.85\ (5820)\ (12)} = 3.30 \times 10^{-6} f_s$$

$$M' = Ta' = f_s A_s \left(2.50 - \frac{k'd}{2}\right)$$
$$= f_s\ (0.196)\quad (2.50 - 1.65 \times 10^{-6} f_s)$$

By equating M to M' theoretical values of total ultimate load, q, can be found for specific assumed values of f_s at failure. The total live load at ultimate capacity can be found by deducting the dead load of the slab, 63 psf, from the total ultimate load. This is done in Table 2.

TABLE 2—MOMENTS AND LOADS AT VARIOUS STEEL STRESSES

Steel stress, f_s, psi	Lever arm, a', in.	Ultimate resisting moment, M', ft-lb	Total ultimate load, q total, psf	Total live load at ultimate, q live load, psf
180,000	2.20	6470	264	201
190,000	2.19	6790	278	215
200,000	2.17	7080	289	226
210,000	2.15	7400	302	239
220,000	2.13	7680	314	251
230,000	2.12	7970	326	263

To obtain a theoretical value for the ultimate load the value of steel stress at failure must be predicted. For a bonded cable this value is normally taken equal to the ultimate strength of the steel. For an unbonded cable this value is generally assumed as 80 to 90 percent of the ultimate steel strength. The slab tested contained unbonded steel cables with an ultimate strength of 250,000 psi, which placed the theoretical ultimate live load between 226 and 257 psf.

The value of ultimate live load obtained experimentally equaled 238 psf which compares favorably with values given above. The crack pattern of the bottom of slab at failure is depicted in Fig. 17. Initially cracks occurred at the centers of each edge and gradually spread toward the center of the slab. Cracks diagonally across the corners of the slab formed during the later stages of loading. Ultimate rupture of the slab occurred gradually and was indicated when further inflation of air bags only gave a drop in air pressure. There was no sudden collapse of the slab. Upon removal of the air bags an inspection of the top surface of the slab showed crushing of the concrete along the transverse centerline. A residual deflection of less than 2 in. at the center of the slab was measured after removal of the load.

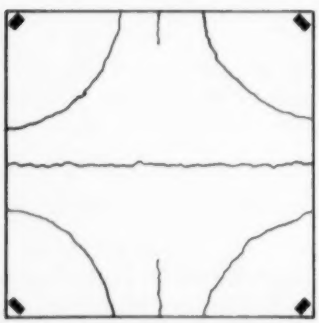

Fig. 17—Actual crack pattern at ultimate load

The deflection at ultimate load was about 4 in. at the center of the slab and averaged about 3 in. at the centers of each edge. If the wires had been entirely free to slip, so that a uniform stress had been obtained in them, the average stress in the wires, based on the above deflections, would have been much less than the 210,000 psi needed to carry an ultimate live load of 238 psf. Apparently the wires were prevented from slipping by frictional forces along their length, thus en-

abling them to develop a much higher stress at midspan.

DESIGN LOAD FOR THE SLAB

It may be interesting to discuss the working load that may be applied to this slab, bearing in mind that at present no building code can be directly applied to the design of a prestressed flat slab. If we proceed on the basis of no permissible tension in concrete, corresponding to a prestress of 418 psi, the moment capacity would be:

$$M = fI/c = fbd^2/6$$
$$= \frac{418 \times 12 \times 5^2}{6 \times 12} = 1740 \text{ ft-lb}$$

Equating this to the edge moment of $0.562\ qa^2$, we have, for $a = 7$ ft:

$$q = \frac{1740}{0.562 \times 7^2} = 63 \text{ psf}$$

which is the dead load of the slab itself. Thus, if tensile stresses are not allowed, no live load can be permitted on the slab at all. The actual test showed excellent behavior of the slab up to at least a live load of 80 psf, with a center deflection of no more than ¼ in. due to live load, although a maximum tensile stress of about 500 psi existed. Thus if the slab was designed for a live load of 80 psf with extreme fiber stresses of about 500 psi, it would still have a factor of safety of three at ultimate failure considering live load only, or a factor of safety of more than two for both dead and live loads. Hence it is evident that the ordinary criterion for design may not be directly applicable to prestressed flat slabs.

CONCLUSIONS AND DISCUSSION

On the basis of the results for the full-sized slab tested, certain conclusions can be qualitatively advanced. Quantitative answers to some questions will have to be based on additional tests. The main conclusions are listed below.

1. The use of plastic air bags for applying a uniform load on a slab was practical and highly convenient.

2. Prior to cracking of the slab, the classical elastic theory can be applied to obtain slab deflections and distribution of bending moments across the slab, as well as the stresses and strains in the concrete. Deflections and strains are necessarily based on some estimated value of the modulus of elasticity as given by cylinder tests.

3. Cracking load for the slab can be predicted by the elastic theory within an accuracy of about 10 percent, using the modulus of rupture obtained from beam specimens provided that the state of stress at the critical point is nominally uniaxial. Deflection and strain readings indicated possible minute cracking of the concrete sooner than can be detected visually by magnifying glasses.

4. The ultimate flexural strength of the slab was developed at a unit stress of about 210,000 psi in the steel wires. This indicated that the so-called unbonded wires were not entirely free to slip along their length. Frictional forces

along the wires apparently limited the amount of slippage and helped to develop a higher stress in the wires.

5. Based on test results, this slab could be designed for a working load of about 80 psf producing a maximum tensile stress of about 500 psi in the concrete. While this amount of tension cannot be a general design criterion for prestressed concrete slabs, it is evident that the concept of allowing no tension in prestressed concrete should be discarded.

While this simple slab represented a fundamental case of the over-all flat slab problem, in practice continuous slabs are more widely adopted, and curved cables rather than straight ones are generally used. Hence it is desirable that more tests be performed using curved cables and continuous slabs. To investigate the ultimate strength of the slabs, it would be desirable to measure stresses and strains in the wires as well as in the concrete. Other problems, such as the modulus of elasticity of concrete after it has been prestressed, the criterion for cracking under a biaxial state of stress, and the amount of friction along the unbonded wires also may need further study if accurate interpretations of test results are to be obtained.

ACKNOWLEDGMENT

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For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Title No. 53-14

Lining of the Tecolote Tunnel*

By E. R. CROCKER†

SYNOPSIS

Special problems encountered in excavating and concrete lining the Tecolote Tunnel near Santa Barbara, Calif., are described. The main problem was inflow of water up to 9000 gal. per min at temperatures which reached 117 F. Methods of overcoming problems, among the most difficult in Bureau of Reclamation construction undertakings, are described, and information on concrete materials, mixes, and placing procedures is presented.

INTRODUCTION

Tecolote Tunnel is the key structure of the Bureau of Reclamation's Cachuma project in California. The tunnel will divert stored flood waters of the Santa Ynez River through the rugged Santa Ynez Mountains to the irrigable lands and municipalities between the mountains and the Pacific Ocean.

A brief description of the local geography will permit a visualization of the scope and purpose of the project. As shown in Fig. 1, contrary to the north and south trend of the California coast, the coastline of Santa Barbara County extends from its boundary with Ventura County in almost a true east and west direction to Point Conception, a distance of about 67 miles. At Point Conception the coastline turns abruptly north and extends generally north about 60 miles to the vicinity of San Luis Obispo. The Santa Ynez mountain range rising to a general elevation of about 3000 ft, roughly parallels the south coastal line at a distance of 5 to 8 miles leaving a strip of flat land and gently sloping foothills from 2 to 5 miles wide for about 15 miles on each side of the city of Santa Barbara. It is for this coastal strip, including the city, that the project will provide a water supply to supplement that pumped from the underlying ground-water basin and other diversion from the Santa Ynez River.

Immediately north of and parallel to the crest of the mountains about 3 miles distant, the Santa Ynez River flows westerly to discharge into the Pacific Ocean at Surf, about 20 miles north of Point Conception.

As a major feature of the Cachuma project, Cachuma Dam has been constructed

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Fig. 1—Location map of Cachuma project

on the Santa Ynez River to intercept and conserve the flood waters of that stream. The reservoir formed by Cachuma Dam has a capacity of 176,000 acre-ft of divertible water and an estimated annual yield of 33,000 acre-ft. Tecolote Tunnel is designed to divert the major portion of this annual yield from the reservoir to the south coastal plain.

TUNNEL DESIGN

Tecolote Tunnel pierces the Santa Ynez range at about 650 ft above sea level and about 2400 ft below the mountain crest at the point of crossing. The Santa Ynez range is composed of rocks of the Miocene, Eocene, and Cretaceous ages, all extensively folded and faulted. One major fault has a stratigraphic displacement of about 12,000 ft. The tunnel is 6.4 miles long, is circular in cross section, and has a diameter of 7 ft inside the concrete lining. The lining has a minimum thickness of 8 in.

The inlet portal of the tunnel is in Cachuma Reservoir 90 ft below water surface elevation at maximum capacity and is provided with a gate shaft having five gates at 18-ft intervals for selective diversion of the reservoir water. The control gates, access to which is provided through a shaft about 125 ft deep, consist of an emergency 30-in. gate valve followed by a 36-in. tube valve for regulating the amount of water diverted. A 48 x 28-in. Venturi meter is provided immediately upstream from the emergency gate. These control works were located 800 ft from the portal to avoid a small drainage course crossing the tunnel line at a ground surface about 40 ft above the top of the tunnel and 43 ft below maximum reservoir water surface.

The tunnel was originally designed to have a horseshoe section 7 ft in diameter inside the lining. For reasons to be presented later, the finished section was changed

to a circle 7 ft in diameter. The 7-ft diameter was selected as the minimum size which would permit installation of anticipated ventilating and unwatering facilities and leave room for haulage and other traffic in the tunnel. As required capacity was only 100 cu ft per sec, the required slope was 0.00025, corresponding to a total fall of 8.4 ft in the length of the tunnel. Since the elevation to which the water could be delivered in the service area was an important consideration, the minimum gradient was adopted.

The design for the tunnel contemplated use of steel H-beam supports where rock was not self-supporting and provided a minimum of 4 in. of concrete between supports and the inside face of the lining. Supports were required for about 98 percent of the length of the tunnel. Normal supports were 4-in. H-beams generally spaced at 5-ft centers.

Bids for construction of the tunnel were opened Dec. 29, 1949, and a contract was awarded to the low bidder, Carl Halvorson, Inc., Portland, Ore., and H. Halvorson, Inc., Spokane, Wash., as joint venturers. During the period of advertisement, prior to the bid opening, some prospective bidders expressed concern over the possible water hazard as indicated by other tunnel construction in the Santa Ynez range, but none apparently visualized the combination of large water inflow and heat that eventually developed.

TUNNEL EXCAVATION

Excavation from the inlet portal began Mar. 3, 1950, and at the outlet portal on May 3, 1950, delay at the latter being occasioned by the necessity for constructing about 2 miles of access road, much of it on rough and precipitous sidehill. Excavation of the tunnel was delayed and hampered by unprecedented conditions and difficulties. It was holed through Jan. 15, 1955, almost 5 years after excavation was started.

In the inlet portion of the tunnel progress was delayed by the presence of methane gas which resulted in one explosion in which 11 men received moderate burns; an inflow of water under high pressure carried several hundred yards of sand into the tunnel; squeezing or swelling ground deformed and displaced long reaches of the steel H-beam supports; and grouting was needed to seal off water in locations where rock was incapable of supporting grout connections, entailing construction of concrete bulkheads for the grouting operations.

In the outlet portion, rock conditions were generally much more favorable, although on several occasions construction of concrete grouting bulkheads was necessary, and on one occasion a zone of broken shale having water under pressure of about 250 psi was encountered. Some 200 cu yd of this broken shale flowed into the tunnel, and progress was halted for many months. Efforts to advance the heading by use of breast boards, timber and steel-rail spiling, and other conventional methods were all unsuccessful, and excavation through this 200-ft reach, in which about 3500 gal. of water per min developed, was accomplished only by excavation of parallel side drifts advanced alternately with the heading. Following advancement through this zone, water in increasing amounts was encountered and in the

next 2400 ft a total of 104 grouting setups were made and 18,500 sacks of cement were injected into the rock seams. These grouting efforts were only partially successful; after excavation the water inflow totaled about 1 gal. per min for each foot of the 2400 ft. In advancing this 2400 ft there was a gradual increase in the water temperature from 87 to 112 F and at the end of this reach, with 4716 ft of unexcavated tunnel remaining, the limits of the contractor's unwatering and ventilating equipment had been reached. Faced with the requirement for supplementing or completely revamping his unwatering and ventilating facilities, the contractor suspended all tunneling operations July 23, 1953.

Work of advancing the heading was resumed Apr. 12, 1954, after the contractor had arranged for an adequate power supply and had installed unwatering facilities adequate for handling 10,000 gal. per min and ventilating equipment capable of providing about 10,000 cu ft of air per min. No attempts were made to grout off water which continued to seep into the tunnel in large amounts, eventually attaining a maximum of about 9000 gal. per min. As the tunnel heading was slowly advanced, the temperature of the inflowing water and the surrounding rock increased from 112 to 117 F in a distance of about 500 ft. While about 10,000 cu ft of air per min was supplied to the heading by the ventilating fan and the four compressors, humidity in the working area was generally about 100 percent and atmospheric temperatures, except at points of discharge of ventilating and exhaust air, approached the temperature of the tunnel walls and inflowing water. Under these conditions, labor efficiency was low and progress on excavation was slow.

However, after reaching a maximum of 117 F the temperature of rock and water started to decline and in the next 1000 ft reached a near normal 90 F. As temperature declined both efficiency and progress improved, and except for the problem of coping with large quantities of water, tunneling operations became routine and continued so to the time of holing through on Jan. 15, 1955.

LINING SCHEDULE

As would be expected, lining operations were affected greatly by the conditions encountered during excavation. About 44 percent of the length of the tunnel was lined from the inlet portal and about 56 percent from the outlet portal. As there was a decided difference in conditions in these two portions, and in methods and procedures of lining, the lining operations are described separately.

When tunnel excavation had been completed to a point 14,900 ft from the inlet portal in March, 1951, it became apparent that closure of the river section of Cachuma Dam would be accomplished before the next flood runoff season, thus introducing the possibility that the inlet portal would be inundated. It was therefore decided to discontinue excavation and line this portion of the excavated tunnel while access from the inlet portal was still possible.

LINING OF INLET PORTION

In consideration of the small amount of cover over the tunnel between the inlet

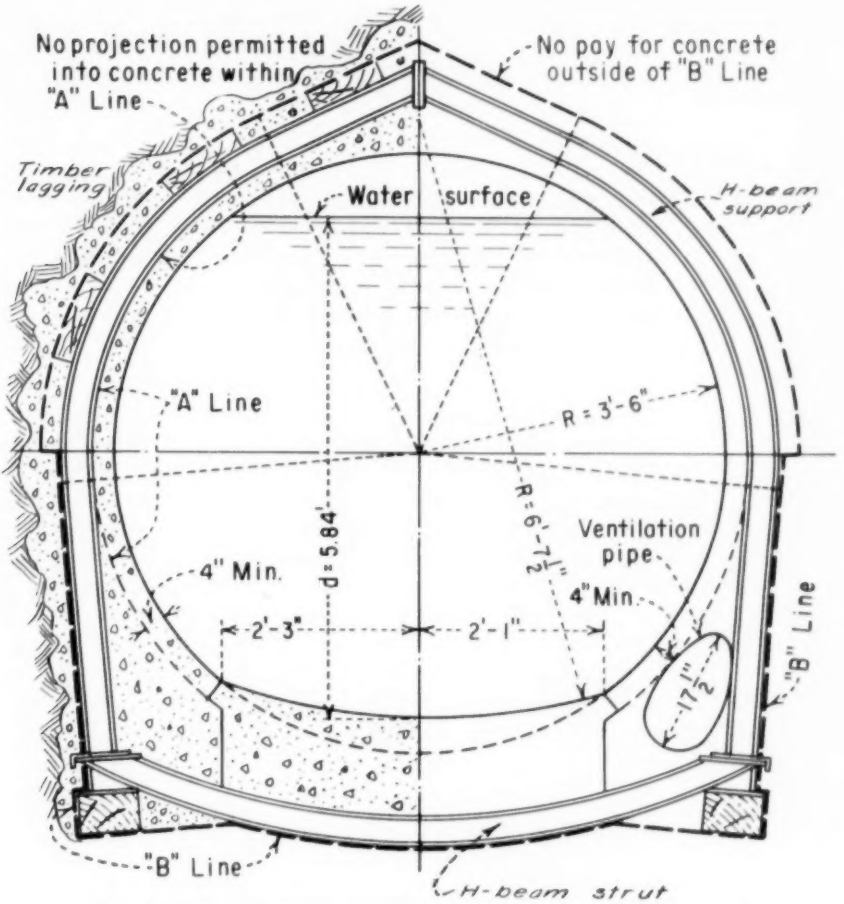

Fig. 2—Principal support and shape of inlet portion of tunnel

portal and the gate shaft, as previously described, and the danger of leakage into the tunnel if the reservoir water should raise to a dangerous level, the 800-ft reach of tunnel between the portal and the gate shaft was lined before plans were perfected for lining the remainder of the inlet portion. Lining this short reach was a routine operation with no special problems involved.

During the last year of excavation in the inlet heading it had been noted that long reaches of innocent-appearing rock, when excavated, had developed swelling or squeezing characteristics and that many of the H-beam steel supports installed had become displaced or distorted by ground pressures. As a result, it was necessary to replace and realign the supports for long reaches of the tunnel, provide additional supports at intermediate spacings, install heavier H-beam sections, and provide struts between the bottoms of the supports, as shown in Fig. 2. Subse-

quently, long reaches of the tunnel had supports spaced as close as 12-in. centers.

By the time all this work had been accomplished, the flood season was imminent, and although there was no certainty that floods in storable amounts would occur, plans for operating through the portal were abandoned and arrangements were made to carry on lining operations through the gate shaft. Accordingly, the batch plant previously erected at the portal was moved to the shaft site and a quick-closing bulkhead for installation at the portal in case of flood was provided. Adjacent to the gate shaft, which is offset from the tunnel centerline, a 24-in. well had been provided from ground surface to tunnel on the tunnel centerline for future ventilation. All concrete placed in the tunnel was dry-batched on the surface and lowered in an elongated hopper through this well to the tunnel where it was dumped directly in dry-batch cars for transportation to the mixer near point of placement.

Ventilation

In considering plans for lining operation, ventilation of the dead-end tunnel presented a major problem. This problem was aggravated by the decision to line the arch and sidewalls of the circular section as a first operation, leaving a width of 4 ft 2 in. in the bottom to be placed after sidewalls and arch had been completely placed. Due to limited working space in the excavated tunnel section, it was thought necessary to remove the 14-in. ventilating pipe back of the concrete placing equipment and it did not appear practicable to carry any type of ventilating pipe into the constricted area occupied by the forms. Also with the ventilating line removed during the first operation, no ventilation would be had during placement of the bottom lining unless the line should later be replaced.

To resolve this difficulty it was decided to change the previously contemplated horseshoe section to a circular section and deform the 14-in. ventilation pipe to an oval section and place it in one of the lower corners of the excavated section where it would be buried in the concrete (see Fig. 2). With the forward or downstream end left open, air could be forced through this deformed pipe and returned to the open gate shaft through the working area. Provision was made for filling the pipe with concrete as placement of the bottom section of lining progressed toward the portal. This plan worked as expected, although the air returning through the placed lining did pick up enough hydrating heat from the concrete to occasion discomfort.

Aggregates and concrete

All aggregates for concrete were required to be furnished by the contractor and to meet the usual U. S. Bureau of Reclamation specifications. Coarse aggregate was required to be separated into two sizes: 3/16 to ¾ in. and ¾ to 1½ in. From previous tests it had been determined that aggregates meeting requirements could be produced from natural material forming the bed of the Santa Ynez River near the inlet portal and this was so stated in the construction specifications. This material while meeting minimum requirements was not first class, contained elements

reactive with cement alkalies, and contained about 15 percent of comparatively weak and lightweight shale and siltstone particles. However, there was no better source of material within 100 miles of the tunnel inlet.

TABLE 1—SAND GRADING REQUIREMENTS

Individual screen size group	Permissible percent each size, by weight retained on screen
No. 4—⅜ in.	0-5
No. 8—No. 4	5-20
No. 16—No. 8	10-20
No. 30—No. 16	10-30
No. 50—No. 30	15-35
No. 100—No. 50	12-20
Pan—No. 100	3-7

The contractor sublet production of concrete aggregates, and a plant was set up in the dry river bed about ¼ mile from the inlet portal. This plant, although old and dilapidated, was equipped with the necessary crushers, screens, and washers for production of material meeting the specifications. Much of the sand occurring in the river bed material was deficient in fines, and considerable quantities were required to be wasted on this account. However, by working in selected areas, the required quantity of acceptable sand was produced. Sand grading requirements are shown in Table 1.

In January, 1952, after most of the aggregate required for the lining being placed from the inlet portal had been produced, a sudden flood in the Santa Ynez River completely wrecked the processing plant. Minor shortages in aggregate quantities previously produced were made up from a commercial plant on the Sisquoc River about 50 miles from the inlet portal.

Although construction specifications required coarse aggregates to be separated into only two sizes, in finish screening at the batch plant it was separated into three sizes—4 to ⅜ in., ⅜ to ¾ in., and ¾ to 1½ in.—for better control of grading.

The mix was proportioned for the concrete to have 28-day compressive strength of 3000 psi. For the sidewalls and arch of most of the inlet leg lining, mix A in Table 2 was used. For invert lining the mix was varied from that for the sidewalls and arch since placement at a lesser slump was possible and desirable. Mix B in Table 2 was used for most of the invert.

Type II, low-alkali cement, was furnished by the government and was trucked 214 miles in bottom-dump bulk tank trucks to the batching plant at the tunnel portal.

Pozzolanic admixture

Construction specifications did not contemplate use of pozzolan in the concrete. However, as a means of further counteracting reactive elements in the aggregate and to provide additional workability, pozzolan in the amount of 20 percent of the originally proposed weight of cement was substituted. This was accomplished under an appropriate order for changes. The pozzolan was produced and ground at Schuman, Calif., about 50 miles from the tunnel. Of peculiar interest is the fact that this pozzolanic material occurs as an oil impregnated shale. It is excavated from an open pit, run through a crusher, screened to the desired particle size, spread in a large mass on the ground to a depth of about 3 ft and

ignited, and the whole mass is calcined without appreciable surface flame. It was found that heat developed in the mass could be controlled within narrow limits by varying the screened particle size. For the pozzolan the desired calcination temperature of 1800 F is attained when the material is between the ⅜- and ½-in. size. After calcination the material was required to be ground to a fineness such that not more than 12 percent would be retained on a No. 325 screen.

Clean-up problems

During excavation of the inlet portion several water inflows of considerable magnitude were encountered, but by the time lining was undertaken they had all decreased so that the total accumulation was about 300 gal. per min. This accumulation was so generally scattered throughout the tunnel that collection in pumpable quantities in any one place was rare. Since the predominating siltstone through which the tunnel was excavated broke down under action of air and water, traffic through the tunnel churned up material on the invert into a soupy mess that rendered bottom cleanup tedious and difficult. This task was aggravated by the closely spaced bottom struts. In preparing the foundation for the base of the sidewalls and placement of sills for the forms, it was possible to sweep or rake much of this mud into the space left between the sills. Even so it was found necessary to pack considerable quantities of gravel into the mud to stabilize it and provide a firm foundation for the sidewall concrete.

Following completion of arch and sidewall lining, the bottom was cleaned for placing the invert. This operation was more tedious and troublesome than clean-up under the sidewalls as the mud had to be picked up and removed from the tunnel (Fig. 3). Here again, the close spacing of the bottom struts prevented use of mechanical equipment or even conventional hand tools. The only practicable method was hand scraping and dipping to the required depth and stabilizing the remainder with gravel (Fig. 4).

Placing procedures

For placing arch and sidewall lining the contractor provided steel forms constructed in sections 10 ft long, hinged at the top and near the bottom of each side so that they could be collapsed and moved forward on a jumbo through forms in use (Fig. 5). The forms were not a full circle but had an open space about 4 ft wide at the bottom (Fig. 6). Bottom edges of side forms were supported on timber sills resting on the bottom of the excavated section. For placing this concrete, the contractor used a pneumatic gun which discharged concrete through a 6 in. slick line at the top of the form (Fig. 7). During placement both external and immersion-type vibrators were used. Except at the start of each successive concrete placement, the end of the slick line was kept well buried in the fresh concrete.

Actual operations involved dry batching materials at the gate shaft and lowering them to the tunnel level in an elongated hopper through the ventilating well as previously described. The hopper discharged into bottom-dump batch cars which transported the material to a ramp over a dry belt leading to the mixer

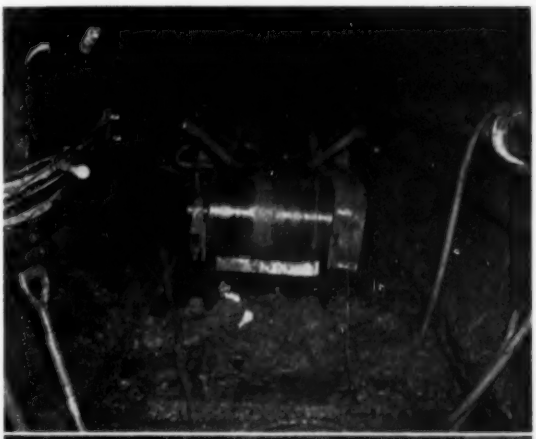

Fig. 3—Reversible conveyor was used to carry mud from invert to muck cars. The mud was replaced with stabilizing gravel followed by concrete

Fig. 4—Invert of inlet portion cleaned of mud and stabilized with gravel before concrete was placed

(Fig. 8, 9, 10). The mixed concrete was discharged onto a wet belt (Fig. 11) which led to a hopper above the placing gun, and from the gun it was placed outside the forms as described above. Original plans contemplated continuous placement, but necessary cleanup did not permit this procedure, and placement was generally on a one or two shift per day basis. Construction joints between successive days' work were formed by constructing vertical bulkheads between the forms and the rock walls of the tunnel.

As the inlet portal was not inundated by reservoir water as anticipated, concrete materials for the invert were batched at the plant at the gate shaft, hauled to the portal in a truck mixer, and mixed en route outside the tunnel. For transportation to the point of placement, a train of three agitator cars (Fig. 12) having a total capacity of 10½ cu yd was arranged so that concrete was discharged forward from one car to another until it was all discharged from the front end of the leading car onto a conveyor belt. The belt carried concrete forward to the tunnel floor where it was vibrated. A weighted steel screed pulled by a tugger anchored

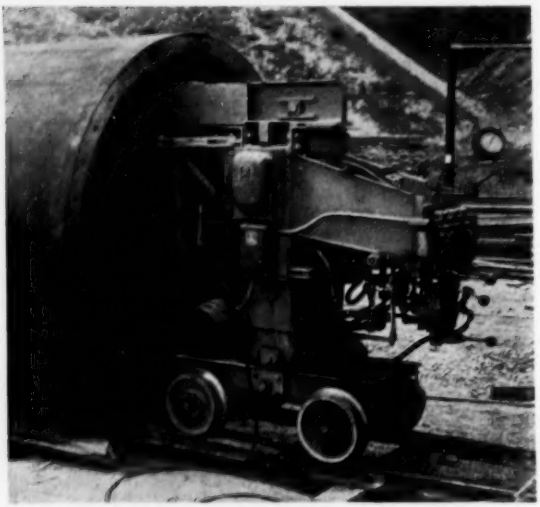

Fig. 5—Operator's end of form carrier. Forms were collapsed and carried forward through forms already in place

to the car track and riding on the bottom edges of the sidewall concrete, shaped the invert concrete to the required circular arc (Fig. 2), after which the surface was hand troweled.

A total of 14,600 ft of tunnel was lined from the inlet portal, involving placement of 18,300 cu yd of concrete. Since specifications provided definite lines to which concrete would be measured for payment, the pay quantity was only 13,685 cu yd, 4615 cu yd being placed in tunnel overbreak.

Placement of lining, exclusive of the 800-ft section between portal and gate shaft, was started on Dec. 22, 1952, and completed Jan. 27, 1954. Placement was not continuous as there was one interruption of 7 weeks during placement of the arch and sidewalls pending a contract adjustment for bottom cleanup in the areas of closely placed bottom struts, and another of 6 weeks between completion of the sidewall and arch lining and start of the placement of the invert lining. There were also many shorter interruptions due to equipment failures and need for plant adjustments. While work was generally prosecuted on a three-shift, 6-day week basis, actual concrete placement was usually confined to about one and one-quarter shifts per day, the remainder of the day being spent in bottom cleanup and moving forms.

LINING OF OUTLET PORTION

Immediately following holing through of the tunnel Jan. 15, 1955, preparation for lining the remaining 18,666 ft from the outlet portal was started. Due to the large quantities of inflowing water and high temperature with its attendant ventilation problem, it was apparent that a procedure different from that followed in the inlet portion would be necessary. The general plan adopted by the con-

Fig. 6—Forms for tunnel lining in position for concrete placing in inlet portion

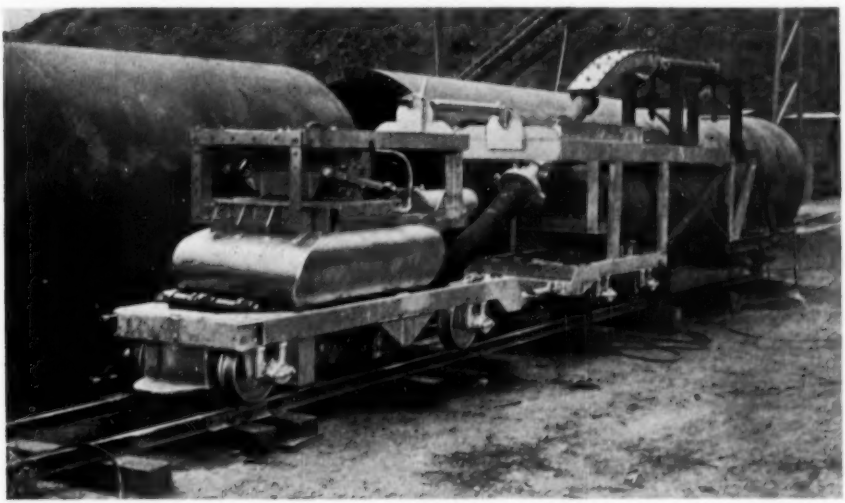

Fig. 7—Pneumatic gun for placing concrete in crown of arch and used in inlet portion of lining

tractor involved use of full circle forms and lining the full tunnel section as one operation. The change from the horseshoe section originally contemplated to the circular section was approved to accommodate the contractor's plans for handling

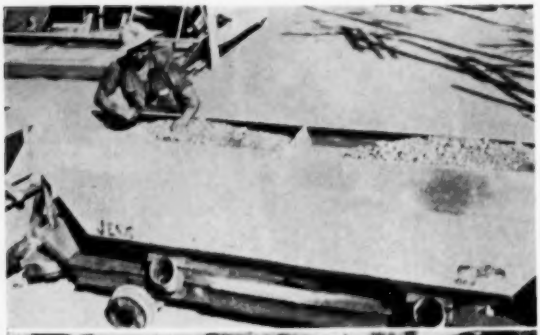

Fig. 8—Two ¾-cu yd batches were carried in these cars to the mixer at the forms in the tunnel

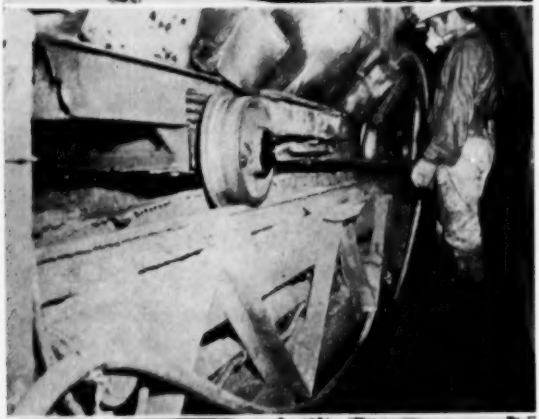

Fig. 9—Batch cars discharged onto this dry belt which carried the materials to the mixer

Fig. 10—Dry-batched materials going up conveyor belt to mixer in tunnel

and disposing of the water as later described and as shown on Fig. 13. The suction headers and the laterals above the floorline were completely embedded in the concrete lining. To protect the concrete lining from the inflowing water during placement, sheet metal panning was placed between the steel H-beam tunnel supports and nailed to the lagging which was installed at spacings such as to prevent buckling of the panning from lateral pressure of the concrete (Fig. 13). This panning trained the water to the invert of the tunnel. The invert was over-excavated

Fig. 11—Conveyor belt carrying concrete from mixer to concrete pump

to depths required to handle the water and refilled with coarse gravel into which water from behind the panning could infiltrate. In the bottom corners of the excavated horseshoe section, one or more 6-in. header or suction pipes were installed with 1½-in. pickup laterals as required. The ends of the laterals were covered with wire mesh strainers extending into the gravel fill in the invert (Fig. 13). By use of vacuum pumps attached to the headers it was possible to keep water below the top of the gravel fill while concrete was being placed.

Eventual disposal of the water as placement of lining progressed downstream was accomplished by installing successive, increasingly higher bulkhead dams in

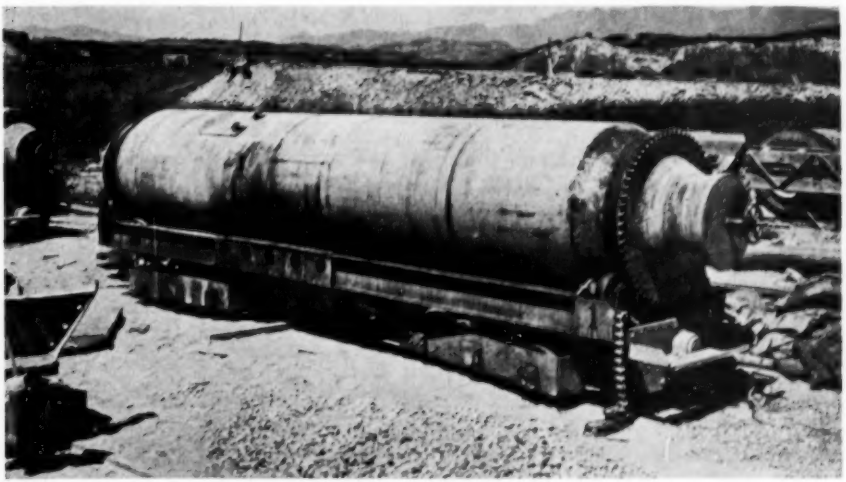

Fig. 12—Agitator cars carried concrete for invert, premixed in a truck mixer at the inlet portal

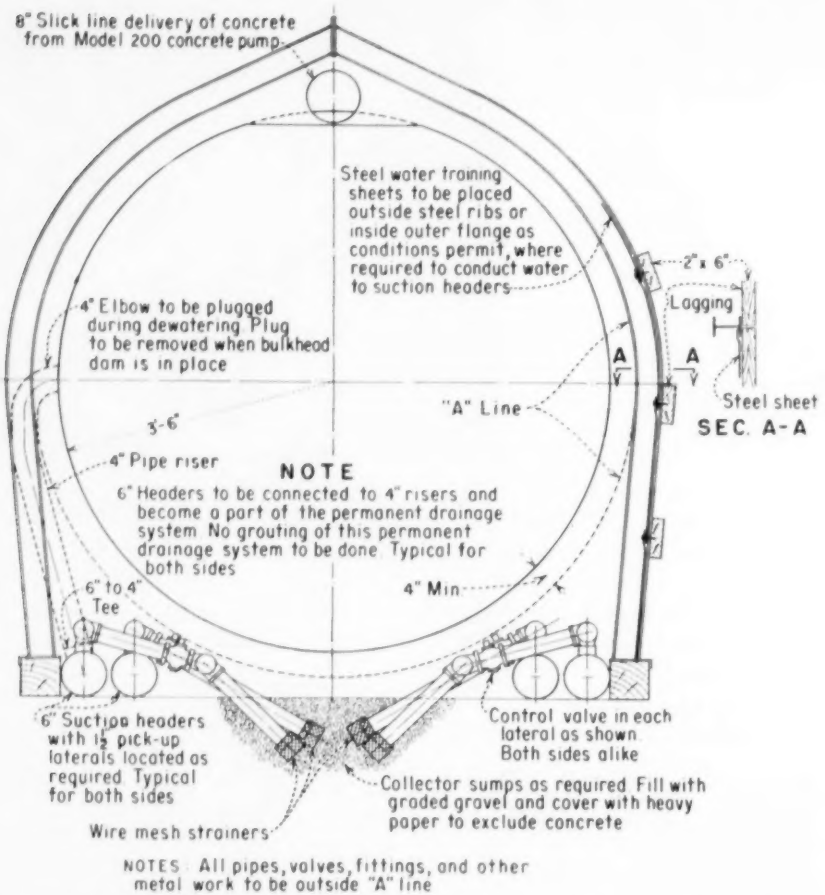
8" Slick line delivery of concrete from Model 200 concrete pump
Steel water training sheets to be placed outside steel ribs or inside outer flange as conditions permit, where required to conduct water to suction headers
2" x 6" Lagging
4" Elbow to be plugged during dewatering. Plug to be removed when bulkhead dam is in place
A
A
"A" Line
Steel sheet
SEC. A-A
3'-6"
4" Pipe riser
NOTE
6" Headers to be connected to 4" risers and become a part of the permanent drainage system. No grouting of this permanent drainage system to be done. Typical for both sides
4" Min
6" to 4" Tee
6" Suction headers with 1½ pick-up laterals located as required. Typical for both sides
Control valve in each lateral as shown. Both sides alike
Collector sumps as required. Fill with graded gravel and cover with heavy paper to exclude concrete
Wire mesh strainers
NOTES: All pipes, valves, fittings, and other metal work to be outside "A" line

Fig. 13—Outlet portion of tunnel section and unwatering system to permit and protect concrete lining

the completed lining, providing risers to about mid-tunnel height from the suction headers at frequent intervals, and equipping the suction headers with shut-off valves at intervals as required (Fig. 13). The risers were plugged at the inside surface line of the concrete lining. As each successive bulkhead dam was installed, the plugs in risers upstream therefrom were removed, the shut-off valve in the header adjacent to the bulkhead was closed, and the water trapped behind the lining would flow through the laterals into the header and thence through the risers into the tunnel upstream of each new bulkhead. The water entering the tunnel upstream of all the bulkheads then flowed naturally out the inlet portal of the tunnel. All headers, laterals, and risers were left embedded in the concrete tunnel lining.

Ventilation

Due to the flat gradient of the tunnel, the first several bulkheads installed were required only to partial tunnel height to force the water out through the inlet portal. To provide more effective ventilation and a supply of cool air, a fan having a capacity of about 10,000 cu ft of air per min was installed at the gate shaft to blow air through the tunnel to and through the working area. The fan and compressors at the outlet portal, also having a combined capacity of about 10,000 cu ft of air per min, were kept in operation and delivered this amount of air through the 14-in. fan line and 6-in. compressed air line to the point of concreting operations. The combined air from these sources then flowed downstream to the outlet portal. These partial height bulkheads were utilized until lining had progressed through practically all of the so-called "hot section," and the amount and temperature of the air supplied was maintained at tolerable, if not completely comfortable, working conditions.

After progressing through the hot section of the tunnel with the lining, full height bulkheads were required and thereafter all ventilation was supplied by the fan and compressors at the outlet portal. The pipes conveying this air necessarily terminated somewhat short of the point of concrete placement. For a time a canvas extension of the fan line was run into the formed area where concrete was being placed, but it so interfered with form movement and placing that its use was discontinued with little deterioration in working conditions.

Aggregates and concrete

The contractor's original plan contemplated placing all concrete from and through the inlet portal. The decision to line the inlet portion prior to holing through the tunnel necessitated a change in these plans and the establishment of a concrete batching plant at the outlet portal. As the entire area on the south side of the mountains within any reasonable distance of the outlet portal was completely devoid of material suitable for aggregates, it was necessary to explore the economy of obtaining aggregate from various remote sources. Also, the possibility of the lining eventually being subjected to high hydrostatic pressures resulted in a decision that concrete having a 28-day strength of 4500 psi should be provided.

For reasons of strength and economy the contractor was instructed to obtain coarse aggregates from a terrace deposit of nonreactive material 66 miles southeast from the outlet portal, and sand from the bed of the Santa Clara River about 46 miles east of the portal. The original specifications as to size separation continued in effect. Both sources of aggregates were commercial plants equipped with complete facilities for processing material and delivering it to the tunnel.

Mix C, Table 2, as originally designed contemplated a 28-day strength of 4500 psi. After about 4000 ft of the tunnel had been lined with this mix and when the beginning of the hot zone had been reached, the mix was modified to mix D to include addition of a calcium lignosulfonate water-reducing and retarding agent*

*This is material PP discussed among other such agents by L. H. Tuthill and W. A. Cordon in "Properties and Uses of Initially Retarded Concrete," ACI JOURNAL, Nov. 1955, *Proc.* V. 52, pp. 273-286.

TABLE 2—CONCRETE MIX DATA

Data	Inlet portion		Outlet portion	
Mix	A, arch	B, invert	C, without*	D, with†
Sand, percent, lb per cu yd	34 945	34 1003	34 1007	34 1012
No. 4-¾ in., percent, lb per cu yd	12 329	12 350	8 245	9 280
⅜-¾ in., percent, lb per cu yd	18 494	18 524	23 702	26 807
¾-1½ in., percent, lb per cu yd	36 988	36 1049	35 1068	31 964
Cement, lb per cu yd	523	468	570	554
Pozzolan, lb per cu yd	131 } 654	96 } 564	102 } 672	100 } 654
Water, lb per cu yd	337	301	298	262
Air-entraining agent, cc	190	225	283	80
Water-reducing agent, lb per cu yd	None	None	None	1.5
$W/(C + P)$	0.515‡	0.52	0.44‡	0.40
Slump, in.	4.7	3.0	3.9	4.2
Entrained air, percent	3.35	3.0	3.1	3.8
28-day strength, psi	3244	3571	4390	5560
Sand source	Santa Ynez River bed		Santa Clara River bed	
Gravel source	Santa Ynez River bed		Grimes Canyon terrace	
Specific gravity	Sand-2.57 Gravel-¾ in., 2.50 ¾ in., 2.53 1½ in., 2.56		Sand-2.54 Gravel-2.63	

*Without water-reducing retarder.
†With water-reducing retarder.
‡Due to larger proportion of crushed material and "flats," and greater absorption, water requirement of mixes A and B was greater than that of mix C.

in the amount of 0.26 percent of cement by weight. This material was added primarily to retard set of the concrete which it was believed might be too greatly accelerated by the high temperatures. In the proportion used, the time of set was delayed about 2 hr. It had the additional property that for a given slump, water requirement was reduced about 10 percent with a corresponding increase in strength of the concrete with the same cement content. It was also found that the material had decided air entrainment properties, and that about two-thirds of the previous dosage of air-entraining agent was not needed. The material was purchased in powdered form, and was dissolved in water and added in appropriate amounts to the dry batches by a mechanical dispenser.

Average data for mixes with and without addition of the water-reducing retarder are shown as mixes C and D in Table 2.

Type II cement, purchased f.o.b. the tunnel outlet portal, and transported 289 miles to the job site in bottom-dump bulk tank trucks, frequently exhibited false set characteristics, but fortunately, due to the reagitation occurring in the concrete pump, no inconvenience in placement resulted.

Placing procedures

The batching plant previously installed at the inlet portal and the gate shaft was moved to the outlet end of the tunnel and erected about 200 ft from the portal. The hauling and placing equipment used in the inlet leg was used in the outlet leg (Fig. 14) with the exception that a concrete pump (Fig. 15) was subsituted for the pneumatic gun (Fig. 7).

As previously stated, full circle forms were used and the full tunnel section was lined as one operation. The forms were fabricated in 10-ft sections and were

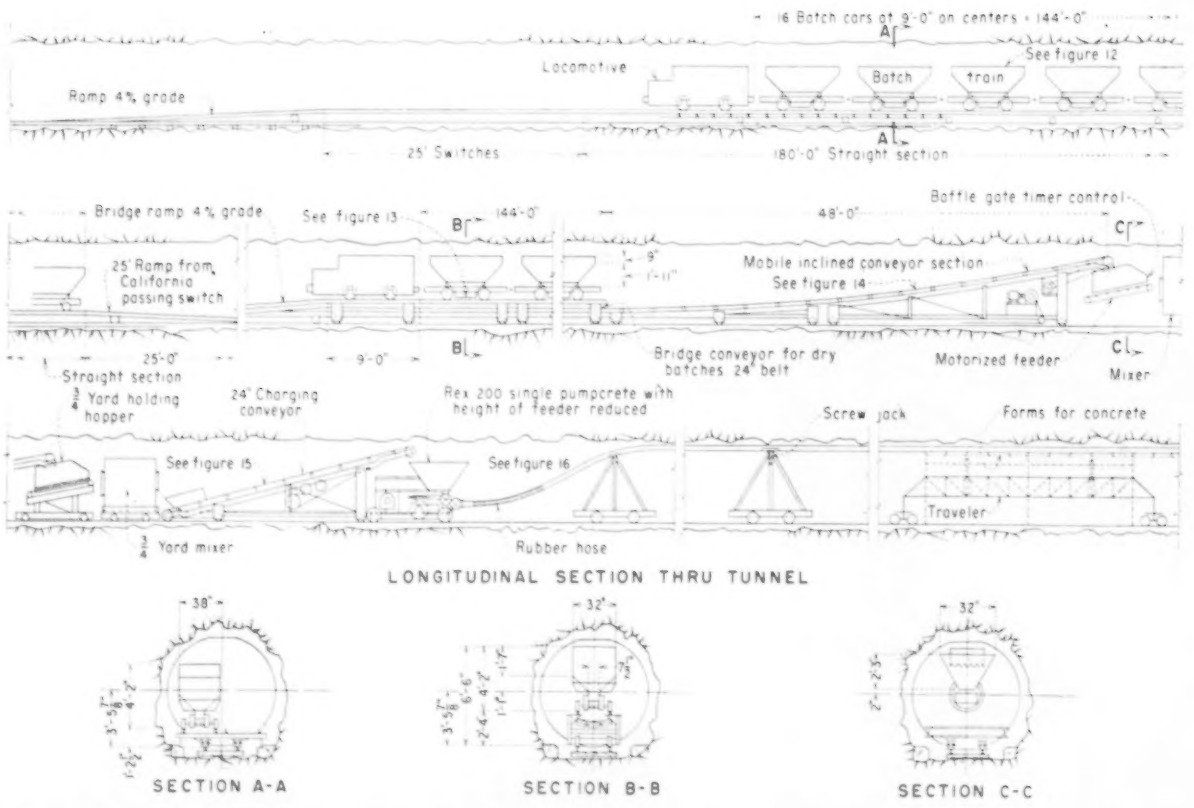

Fig. 14—Arrangement of concrete equipment for tunnel lining—outlet leg. Similar to inlet leg except for pump

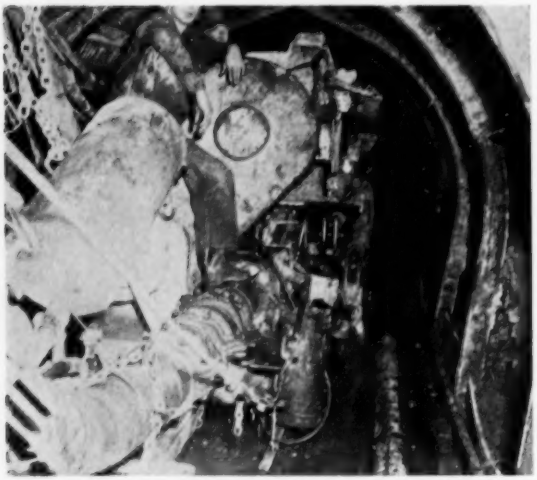

Fig. 15—Concrete pump placing concrete in crown of arch forms through 8-in. discharge line at left

hinged for collapsing and moving forward through the forms in use by the form moving jumbo (Fig. 16). A total of 18 windows was provided in each form section for inspection. Internal vibration and hinged bottom skirts that could be raised for finishing the invert were also provided. In practice, however, these skirts were never lifted as it was found possible to fill the invert completely with concrete with only an occasional shallow air bubble or bug hole. A total of 220 ft of forms was provided. To provide room for an 8-in. slick line between the forms and the steel tunnel supports, the top of the forms was flattened to a chord having a middle ordinate of 2½ in. (see Fig. 13).

In placing the lining, concrete was pumped through the slick line laid above the center of the forms (see Fig. 13, 14, 15) the end of which, except at the start each day, was kept immersed about 5 ft in the fresh concrete. Concrete as discharged was allowed to assume its natural slope behind the forms, usually about 3 to 1. Immersion vibrators were used through the windows and care was taken to vibrate the invert concrete thoroughly so that it completely closed against the bottom of the forms. External form vibrators were also used effectively as concrete in the top portion of the section was being placed. In general, lining surfaces below the horizontal centerline where air holes are normally expected were remarkably free from these defects.

Concrete placement was usually performed during about 12 hr of each day, from 150 to 180 ft of lining being the usual production. During the other 12 hr the bottom of the tunnel was cleaned up for the succeeding day's placement, lagging and collar braces were removed and additional forms were moved forward and set. Joints between succeeding days' placements were on the natural slope of the concrete, except in the invert where a low bulkhead was provided to eliminate feather edges. To start each day's placement, the joint surface was given a coating of cement and sand grout, and as a result it was usually difficult to detect lines between successive days' work.

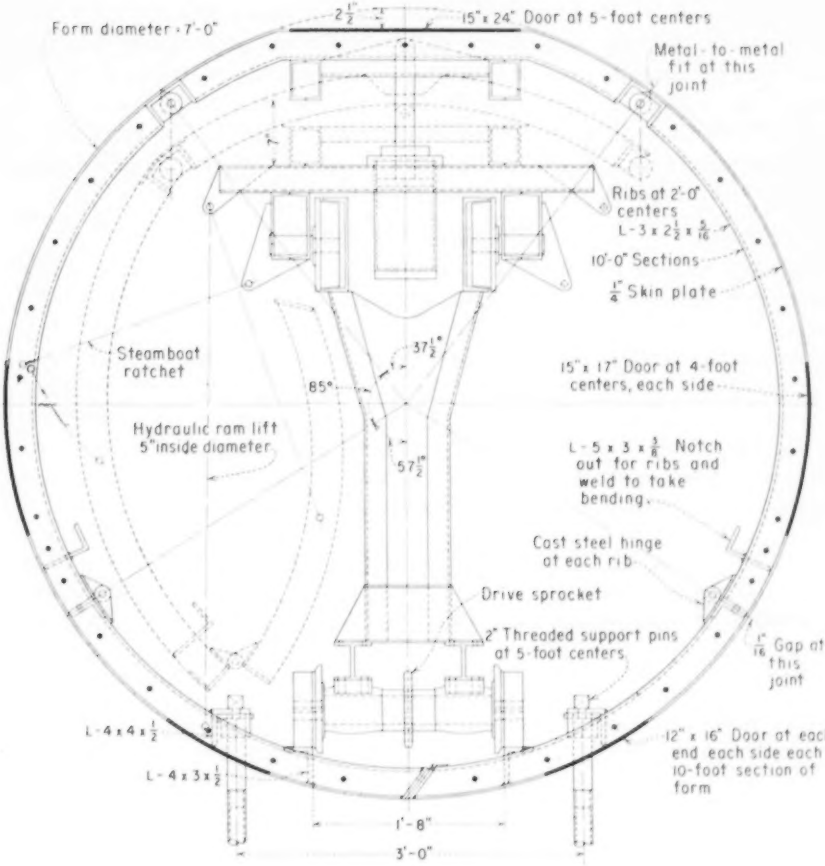
Form diameter - 7'-0"
15" x 24" Door at 5-foot centers
Metal-to-metal fit at this joint
Ribs at 2'-0" centers
L-3 x 2½ x 5/16
10'-0" Sections
¼" Skin plate
Steamboat ratchet
85°
37½°
57½°
15" x 17" Door at 4-foot centers, each side
Hydraulic ram lift 5" inside diameter
L-5 x 3 x ⅜ Notch out for ribs and weld to take bending.
Cast steel hinge at each rib
Drive sprocket
2" Threaded support pins at 5-foot centers
1/16" Gap at this joint
L-4 x 4 x ½
L-4 x 3 x ½
12" x 16" Door at each end each side each 10-foot section of form
1'-8"
3'-0"

Fig. 16—Forms used for outlet portion of tunnel lining

Upon removal of forms, the infrequent rock pockets were chipped out and dry packed with concrete and the entire surface was coated with a paraffin emulsion, white-pigmented sealing compound. This material was selected by the contractor in preference to regular concrete curing preparations which develop a disagreeable odor in confined areas.

Placement of concrete lining in the outlet leg was started on Mar. 14, 1955, and was completed on Jan. 28, 1956. The total pay yardage for the 18,666 ft of lining in the outlet leg was 16,005 cu yd, but due to overbreak and the shape of the section required to be excavated, the total yardage of concrete placed was about 25 percent in excess of that amount.

Although the lining operations in Tecolote Tunnel presented many initial difficulties, solutions for the problems were found. Handling and disposal of the

large quantities of inflowing water were accomplished without sacrifice of the adequacy of the lining. A high quality of concrete was maintained. Perhaps the most memorable experience was the progress through the hottest zone of the tunnel. Here efficiency of the workmen was at low ebb, but turnover of labor was light, each individual apparently considering the condition of heat and humidity a challenge to his stamina. It is certain that in years to come many will be proud to say, "I helped line Tecolote Tunnel."

ACKNOWLEDGMENTS

Tecolote Tunnel was constructed for the U. S. Bureau of Reclamation under the general supervision of L. N. McClellan, assistant commissioner and chief engineer. Grant Bloodgood, associate chief engineer, represented him on much of the supervision and contract negotiations. E. R. Crocker, project manager, was in direct charge of the work, assisted by R. E. Burnett, chief of the construction division of the Cachuma project, and Max T. Hedges, field engineer. Carl M. Halvorson, Inc., and H. Halvorson, Inc., were the contractors. Peter Kiewit Sons' Co. and Coker Construction Co. were subcontractors on the excavation of the last 4716 ft of tunnel and the lining of 18,666 ft in the outlet leg.

For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Title No. 53-15

Theory of the Stresses Induced in Reinforced Concrete by Applied Two-Dimensional Stress*

By BRUCE H. FALCONER†

SYNOPSIS

This paper presents a theory of the stresses which are induced in concrete, reinforced in up to two directions, when under applied two-dimensional stress. It is based on convenient simplifying assumptions of the modes of failure. These assumptions are that failure will occur when either redistributions of internal stress cannot relieve tensions greater than yield in the reinforcement, or when induced compressions in the concrete exceed the ultimate compressive strengths of plain concrete.

Although the actual stresses within reinforced concrete are, in general, indeterminate under given loadings, it is shown that the stresses lie within computable regions of magnitude. Consequently, the theory can be used for the computation of quantities and orientations of reinforcement which are consistent with desired ultimate strengths. This application should prove useful in planning the reinforcement of shear walls, deep beams, and normal beams.

As a particular and illustrative case of the theory, consideration is given to the reinforcement of concrete to carry shear plus axial load. An explanation is offered for the experimentally observed high shear strengths of lightly stirruped beams.

In an appendix the theory is applied to a consideration of the strengths of normal or "shallow" reinforced concrete beams, in which the longitudinal reinforcement is customarily located only near the top and bottom of the cross sections. The theory, as applied, purports to predict the shear strengths of "ideally reinforced" beams with given percentages of web reinforcement, presuming that the reinforcement has marked yield stresses. A comparison is given with the results of tests conducted at the University of Illinois.

INTRODUCTION

The provision of tensile reinforcement to enable concrete to resist applied two-dimensional stress has long been both a theoretical and practical problem, mainly because the reinforcement itself makes the composite material nonhomogeneous,

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and because actual distributions of internal stress may vary with the magnitude of a given applied loading. While there is a vast fund of knowledge on the behavior of concrete components under pure bending or pure axial compression, there is little detailed experimental or theoretical evidence available for structural components under the more complex two-dimensional loadings. With the increasing use of deep beams and shear walls, where stresses may be high, designers are becoming increasingly aware of this lack of knowledge.

The simplest conventional approach in the design of concrete structures is to first compute the induced stresses on the basis of either elementary or elasticity theories for an idealized material, and then to provide reinforcement to carry the tensile components of stress, while presuming the concrete continues to carry, without change, the associated theoretical compressive components of stress. It is usual also "to allow the concrete to carry" unaided any tensile components of stress which may be directly associated with shear, and are within safe working limits of the ultimate tensile stress for plain concrete.

This approach will in general result in soundly designed concrete structures, although there is experimental evidence that for particular loading conditions, symptoms of compressive failure can occur at an unexpectedly early stage.

Other experiments[1,2,3] have shown that this approach is sometimes needlessly conservative and that economies can be made in the provision of certain types of reinforcement—for instance, in the stirruping of beams to resist shear. This knowledge has been reflected in various building codes, principally in the North American continent, which allow a designer to presume that concrete continues to resist a shear component equal in magnitude to the safe tensile working stress, so that stirrups need be provided for only the excess shear.

The ultimate shear strengths of beams can be appreciably higher than even the amended codes would suggest. This has been reported in numerous investigations during the past half-century. However, the author has found no satisfactory theoretical explanation of the phenomenon in the literature. Two design methods have been suggested by previous investigators who have taken advantage of the known increased strength. In 1924, Faber[1] presented his "inclined compression theory;" in 1945, Moretto[4] formulated an empirical equation for the shear strength which takes account of the quantity of longitudinal reinforcement in addition to both stirrups and concrete tensile strength. Wilby[2] has suggested that both these design methods can greatly overestimate the shearing strengths of beams.

It is hoped that the present paper will provide a satisfactory theoretical explanation of the internal stress distribution in reinforced concrete under two-dimensional stress, and also give an explanation of the observed high shear strengths of lightly-stirruped beams.

ASSUMPTIONS

In what follows, it is assumed that two component stress distributions are induced in heavily stressed reinforced concrete—one in the concrete and the other in the reinforcement. The resultant of the two distributions is statically equivalent

to the applied stress field. This statement is consistent with the laws of mechanics and the requirement that the applied stress field must be capable of expression in terms of only normal stresses on principal orthogonal planes. It is of the greatest significance to note from these presumptions that while the applied stress field must act normally on a principal plane, the two component distributions, considered separately, need not do so.

It is assumed that reinforced concrete will fail under an increase of load either when redistributions of internal stress can no longer relieve tensions greater than yield in the reinforcement, or when induced compressions in the concrete continue to exceed the ultimate values for plain concrete. The concrete is presumed able to carry tensile components of stress up to the ultimate tensile stress for plain concrete, but once the linear strains associated with this stress have been exceeded the concrete is considered to be entirely relieved of any tensile stress.

Notation

$\sigma_x, \sigma_y, \tau_{xy}$ = components of the applied stress field
σ_1, σ_2 = induced direct stress components in the reinforcement, expressed in terms of concrete area
σ_c = induced compression in the concrete
$T_{1x}, T_{2x}, T_{1y}, T_{2y}$ = forces in the reinforcement on x and y faces of an element
C_x, C_y = forces in the concrete on x and y faces of an element
α, χ, ϕ = angles defined in Fig. 2
$(\sigma_1)_u, (\sigma_2)_u, (\sigma_c)_u$ = ultimate or limiting values of $\sigma_1, \sigma_2, \sigma_c$
β = defined in Eq. (32)
M = bending moment
S = shear force
jd = effective lever arm of a beam

ANALYSIS

The analysis is developed by considering a reference element of composite concrete and reinforcement under the action of a uniform stress field. The selected element is considered to be sufficiently large for the influence of cracking to have a statistically average effect, yet not so large that a reinforcing rod traversing the element can have other than sensibly constant stress.

Fig. 1 shows the selected element under the actions of the applied uniform stress field. Fig. 2 shows the same element under the component actions of the component materials. Reinforcement is considered placed in two orientations making angles of α and ϕ respectively with the y axis; χ is an unknown orientation of induced unidirectional compression in the concrete. Should the concrete be required to carry two-dimensional compression, it can be shown that tensile reinforcement is unnecessary.

For statical equivalence of the forces on the y face, one may write,

$$-T_{1y} \sin \alpha + C_y \sin \chi + T_{2y} \sin \phi = \tau_{xy} \quad \ldots\ldots (1)$$

$$T_{1y} \cos \alpha - C_y \cos \chi + T_{2y} \cos \phi = \sigma_y \quad \ldots\ldots (2)$$

and similarly for the x face

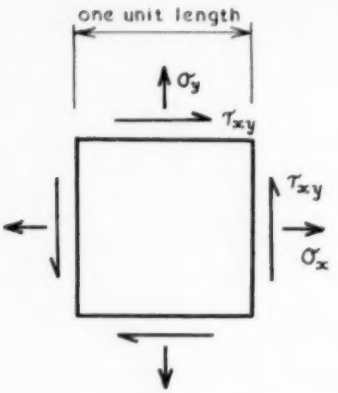

Fig. 1—Element under the actions of applied uniform stress field

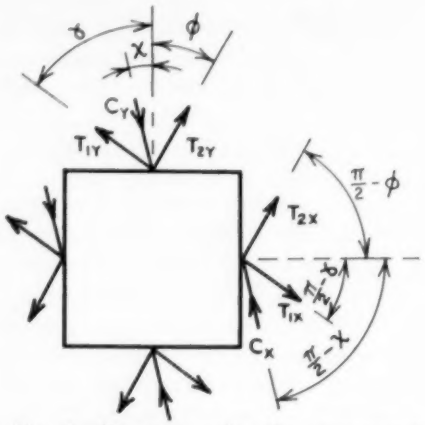

Fig. 2—Element under the component actions of the component materials

$$-T_{1x} \cos \alpha + C_x \cos \chi + T_{2x} \cos \phi = \tau_{xy} \quad (3)$$
$$T_{1x} \sin \alpha - C_x \sin \chi + T_{2x} \sin \phi = \sigma_x \quad (4)$$

It is now convenient to express T and C in terms of direct stresses per unit area of concrete. These are

$$T_{1y} = \sigma_1 \cos \alpha, \quad T_{2y} = \sigma_2 \cos \phi, \quad C_y = \sigma_c \cos \chi$$
$$T_{1x} = \sigma_1 \sin \alpha, \quad T_{2x} = \sigma_2 \sin \phi, \quad C_x = \sigma_c \sin \chi$$

By substitution of these relations in Eq. (1) to (4)

$$\sigma_1 \cos^2 \alpha - \sigma_c \cos^2 \chi + \sigma_2 \cos^2 \phi = \sigma_y \quad (5)$$
$$-\sigma_1 \sin \alpha \cos \alpha + \sigma_c \sin \chi \cos \chi + \sigma_2 \sin \phi \cos \phi = \tau_{xy} \quad (6)$$
$$\sigma_1 \sin^2 \alpha - \sigma_c \sin^2 \chi + \sigma_2 \sin^2 \phi = \sigma_x \quad (7)$$

It will be noted that Eq. (1) and (3) both reduce to Eq. (6). Addition of Eq. (5) and (7) gives

$$\sigma_1 - \sigma_c + \sigma_2 = \sigma_x + \sigma_y \quad (8)$$

or

$$-\sigma_c = (\sigma_x + \sigma_y) - (\sigma_1 + \sigma_2) \quad (9)$$

Substitution of Eq. (9) in each of Eq. (5) to (7) gives

$$\sigma_1(\cos^2 \alpha - \cos^2 \chi) + \sigma_2(\cos^2 \phi - \cos^2 \chi) + \sigma_x \cos^2 \chi - \sigma_y \sin^2 \chi = 0 \quad (10)$$
$$\sigma_1(\sin 2\alpha - \sin 2\chi) - \sigma_2(\sin 2\phi + \sin 2\chi) + (\sigma_x + \sigma_y) \sin 2\chi + 2\tau_{xy} = 0 \quad (11)$$
$$\sigma_1(\sin^2 \alpha - \sin^2 \chi) + \sigma_2(\sin^2 \phi - \sin^2 \chi) - \sigma_x \cos^2 \chi + \sigma_y \sin^2 \chi = 0 \quad (12)$$

Subtraction of Eq. (12) from Eq. (10) gives

$$\sigma_1(\cos 2\alpha - \cos 2\chi) + \sigma_2(\cos 2\phi - \cos 2\chi) + \sigma_x(\cos 2\chi + 1) + \sigma_y(\cos 2\chi - 1) = 0 \quad (13)$$

A recapitulation of Eq. (11), (13), and (8) is

$$\sigma_1(\sin 2\alpha - \sin 2\chi) - \sigma_2(\sin 2\phi + \sin 2\chi) + (\sigma_x + \sigma_y) \sin 2\chi + 2\tau_{xy} = 0 \quad (14)$$
$$\sigma_1(\cos 2\alpha - \cos 2\chi) + \sigma_2(\cos 2\phi - \cos 2\chi) + (\sigma_x + \sigma_y) \cos 2\chi + (\sigma_x - \sigma_y) = 0 \quad (15)$$
$$\sigma_1 + \sigma_2 - \sigma_c - \sigma_x - \sigma_y = 0 \quad (16)$$

By definition α, ϕ, σ_x, σ_y, and τ_{xy} have known values. There are only the three equations, Eq. (14) to (16), for the four unknowns σ_1, σ_2, σ_c, and χ. This shows

that in the general case the problem of internal stress distribution is indeterminate in one degree. Hence to determine the values of the unknowns precisely, one further restrictive criterion must be considered. The first criterion considered was an equation of internal and external work for the element. However, it was abandoned because of the complexity of the computation involved.

GENERAL CASE FOR ULTIMATE LOADS

The basis of the present theory is a presumption that failure will occur when either σ_1 or σ_2, or σ_c exceeds its limiting value. This can be written as

$$0 < \sigma_1 < (\sigma_1)_u, \quad 0 < \sigma_2 < (\sigma_2)_u, \quad 0 < \sigma_c < (\sigma_c)_u$$

Obviously then, values of, say $\sigma_1 = (\sigma_1)_u$ (or of σ_2 or of σ_c) can be assumed and the values of σ_2, σ_c, and χ compatible with it can be computed from Eq. (14) to (16). By this means the regions of σ_1, σ_2, σ_c, and χ compatible with given ultimate strengths can be computed. This is the main result which it is desired to show in the present paper.

SOME PARTICULAR CASES

Simplifications of Eq. (14) to (16) can be written for particular cases. For instance, it is frequently desired for practical convenience to place reinforcement in orthogonal directions. Sometimes it may be convenient to presume, judiciously, the orientation of the induced compressive stress in the concrete.

Orthogonal reinforcement

(a) If the reinforcement is placed orthogonally $\alpha + \phi = \pi/2$, so that $\sin 2\alpha = \sin 2\phi$, and $\cos 2\alpha = -\cos 2\phi$, substitution in Eq. (14) to (16) gives

$$\left.\begin{aligned} \sigma_1(\sin 2\phi - \sin 2\chi) - \sigma_2(\sin 2\phi + \sin 2\chi) + (\sigma_x + \sigma_y)\sin 2\chi + 2\tau_{xy} = 0 \\ \sigma_1(-\cos 2\phi - \cos 2\chi) + \sigma_2(\cos 2\phi - \cos 2\chi) + (\sigma_x + \sigma_y)\cos 2\chi + (\sigma_x + \sigma_y) = 0 \\ \sigma_1 + \sigma_2 - \sigma_c - \sigma_x - \sigma_y = 0 \end{aligned}\right\} \quad (17)$$

(b) $\chi = 45$ deg—If also $\chi = 45$ deg Eqs. (17) become

$$\left.\begin{aligned} (\sigma_1 - \sigma_2)\sin 2\phi - (\sigma_1 + \sigma_2) + \sigma_x + \sigma_y + 2\tau_{xy} = 0 \\ (\sigma_1 - \sigma_2)\cos 2\phi - (\sigma_x + \sigma_y) = 0 \\ \sigma_1 + \sigma_2 - \sigma_c - \sigma_x - \sigma_y = 0 \end{aligned}\right\} \quad (18)$$

(c) $\chi = 45$ deg and $\phi = 0$—If $\chi = 45$ deg and $\phi = 0$ Eqs. (17) reduce to

$$\left.\begin{aligned} \sigma_1 &= \sigma_x + \tau_{xy} \\ \sigma_2 &= \sigma_y + \tau_{xy} \\ \sigma_c &= 2\tau_{xy}, \text{ or } \sigma_1 + \sigma_2 - \sigma_x - \sigma_y \end{aligned}\right\} \quad (19)$$

The equations in the form of either Eq. (18) or (19) can be useful for determining the reinforcement for highly stressed concrete walls, where the design stresses are often computed, and the reinforcement placed, in convenient orthogonal directions. It should be noted of course that the analysis has been derived for tensile values of σ_1 and σ_2, so that no reinforcement is theoretically necessary in the appropriate directions when σ_1 or σ_2 computed from Eq. (18) or (19) is compressive in value.

This simplified approach will often be found convenient for design. It should be realized, however, that greater economies in the use of reinforcement may be justified by using the more general approach.

Reinforcement so that concrete stress has a principal orientation

If χ is also the angle for principal compressive stress of the applied field, σ_x, σ_y, and τ_{xy}, it can be shown from the theory of elasticity that

$$\frac{\tau_{xy}}{\sigma_x - \sigma_y} = \tfrac{1}{2} \tan 2\chi \quad \ldots\ldots (20)$$

The substitution of Eq. (16) in Eq. (14) and (15) gives

$$\sigma_1 \sin 2\alpha - \sigma_2 \sin 2\phi - \sigma_c \sin 2\chi + 2\tau_{xy} = 0$$
$$\sigma_1 \cos 2\alpha + \sigma_2 \cos 2\phi - \sigma_c \cos 2\chi + (\sigma_x - \sigma_y) = 0$$

which is, by rearrangement and division

$$\tan 2\chi = \frac{\sigma_1 \sin 2\alpha - \sigma_2 \sin 2\phi + 2\tau_{xy}}{\sigma_1 \cos 2\alpha + \sigma_2 \cos 2\phi + \sigma_x - \sigma_y}$$

so that

$$\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_1 \sin 2\alpha - \sigma_2 \sin 2\phi}{\sigma_1 \cos 2\alpha + \sigma_2 \cos 2\phi} \quad \ldots\ldots (21)$$

Eq. (21) or (20), together with Eq. (14), (15), and (16), enables σ_1, σ_2, σ_c, and χ to be determined.

Orthogonal reinforcement—If, also, the reinforcement is placed orthogonally, Eq. (21) reduces to

$$\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \tan 2\alpha$$

This result shows that the reinforcement also must be placed in an orthogonal principal direction or the desired orientation of χ will not occur. This is mainly of academic interest. The effect of slight errors in orientation of main reinforcement should not be of much consequence as it is customary in construction to place some distribution reinforcement transverse to the main reinforcement, whether or not this is considered theoretically necessary. The effect of larger errors in orientation can be estimated easily.

PARTICULAR CASE OF COMBINED SHEAR AND AXIAL STRESS

As a particular case of the theory, a loading of combined shear and unidirectional stress will now be considered. One of the sets of reinforcement is considered orientated in the direction of the axial stress component. These conditions can be written as $\sigma_y = 0$ and $\alpha = 90$ deg. Substituting them in Eq. (14), (15), and (16) gives

$$(\sigma_x - \sigma_1) \sin 2\chi - \sigma_2(\sin 2\phi + \sin 2\chi) + 2\tau_{xy} = 0 \quad \ldots\ldots (22)$$
$$(\sigma_x - \sigma_1)(1 + \cos 2\chi) + \sigma_2 (\cos 2\phi - \cos 2\chi) = 0 \quad \ldots\ldots (23)$$
$$-(\sigma_x - \sigma_1) + \sigma_2 - \sigma_c = 0 \quad \ldots\ldots (24)$$

It will be seen that σ_x and σ_1 are directly related. From Eq. (23)

$$(\sigma_x - \sigma_1) = -\sigma_2 \left(\frac{\cos 2\phi - \cos 2\chi}{1 + \cos 2\chi} \right)$$

By substitution of this relation in Eq. (22)

$$-\sigma_2\left(\frac{\cos 2\phi - \cos 2\chi}{1 + \cos 2\chi}\right)\sin 2\chi - \sigma_2(\sin 2\phi + \sin 2\chi) + 2\tau_{xy} = 0$$

which reduces to

$$\frac{\sigma_2}{\tau_{xy}} = \frac{2}{\sin 2\phi + (1 + \cos 2\phi)\tan\chi} \qquad (25)$$

Substitution of Eq. (24) in Eq. (23) gives

$$\cos 2\chi = \frac{\sigma_2}{\sigma_c}(1 + \cos 2\phi) - 1$$

It is a trigonometrical relation that

$$\tan\chi = \sqrt{\frac{1 - \cos 2\chi}{1 + \cos 2\chi}} = \sqrt{\frac{2 - \frac{\sigma_2}{\sigma_c}(1 + \cos 2\phi)}{\frac{\sigma_2}{\sigma_c}(1 + \cos 2\phi)}} \qquad (26)$$

Substitution of Eq. (26) in Eq. (25) gives

$$\frac{\sigma_2}{\tau_{xy}} = \frac{2}{\sin 2\phi + \sqrt{\left[\frac{2 - \frac{\sigma_2}{\sigma_c}(1 + \cos 2\phi)}{\frac{\sigma_2}{\sigma_c}}\right](1 + \cos 2\phi)}}$$

which may be rewritten as

$$\left[\frac{2\tau_{xy}}{\sigma_2} - \sin 2\phi\right]^2 \frac{\sigma_2}{\sigma_c} = \left[2 - \frac{\sigma_2}{\sigma_c}(1 + \cos 2\phi)\right](1 + \cos 2\phi) \qquad (27)$$

The ratio τ_{xy}/σ_c can be computed from Eq. (27) for given values of σ_2/σ_c.

It will be noticed, for instance, that two possible sets of values satisfying Eq. (27) and (24) are

$\sigma_2 = \tau_{xy}$, $\sigma_c = 2\tau_{xy}$, $\phi = 0$, $\sigma_1 = \sigma_x + \tau_{xy}$

and

$\sigma_2 = \tau_{xy}$, $\sigma_c = \tau_{xy}$, $\phi = 45$ deg, $\sigma_1 = \sigma_x$

and these are identical with results based on conventional presumptions, when no allowance is made for the "apparent shear resistance of concrete."

Ultimate strengths—However, if in Eq. (27) σ_c is taken as the limiting value, *i.e.*, $\sigma_c = (\sigma_c)_u$, then the equation, which becomes

$$\left[\frac{2\tau_{xy}}{\sigma_2} - \sin 2\phi\right]^2 \frac{\sigma_2}{(\sigma_c)_u} = \left[2 - \frac{\sigma_2}{(\sigma_c)_u}(1 + \cos 2\phi)\right](1 + \cos 2\phi) \qquad (28)$$

can be used for computing specific values of $\tau_{xy}/(\sigma_c)_u$ and of $\sigma_2/(\sigma_c)_u$. The stress σ_1 can then be obtained from the relation

$$\frac{\sigma_1}{(\sigma_c)_u} = \frac{\sigma_x}{(\sigma_c)_u} + \left[1 - \frac{\sigma_2}{(\sigma_c)_u}\right] \qquad (29)$$

This shows that when $\sigma_2/(\sigma_c)_u$ is small $\sigma_1/(\sigma_c)_u$ can be comparatively large.

Ultimate strength when $\phi = 0$—Substitution of $\phi = 0$ in Eq. (28) gives

$$\left[\frac{\tau_{xy}}{(\sigma_c)_u}\right]^2 = \frac{\sigma_2}{(\sigma_c)_u}\left[1 - \frac{\sigma_2}{(\sigma_c)_u}\right] \qquad (30)$$

Ultimate strength when $\phi = 45$ deg—Substitution of $\phi = 45$ deg in Eq. (28) gives

$$2\frac{\tau_{xy}}{(\sigma_c)_u}\left[\frac{\tau_{xy}}{(\sigma_c)_u} - \frac{\sigma_2}{(\sigma_c)_u}\right] = \frac{\sigma_2}{(\sigma_c)_u}\left[1 - \frac{\sigma_2}{(\sigma_c)_u}\right] \quad \text{.....(31)}$$

Desired ratio of stresses—If it should be desired to plan the reinforcement so that the stresses in the reinforcement have a desired ratio, *e.g.*,

$$\frac{\sigma_1 - \sigma_x}{\sigma_2} = \beta$$

then from Eq. (24)

$$\sigma_2 = \frac{\sigma_c}{1 + \beta} \quad \text{..... (32)}$$

Substitution of Eq. (32) in Eq. (28) gives

$$\left[\frac{2\tau_{xy}(1 + \beta)}{\sigma_c} - \sin 2\phi\right]^2 = [2(1 + \beta) - (1 + \cos 2\phi)](1 + \cos 2\phi) \quad \text{.....(33)}$$

This enables σ_2, σ_c, and χ to be determined uniquely.

DISCUSSION

In its general form, the theory enables the computation of sets of values of stresses which are compatible with equilibrium. No attempt has been made to show the influence of elastic properties of the component materials, as it was felt that the computation would be unwarrantedly complex, and that there would be a doubt of the validity of the computation after redistributions of stress associated with yielding.

In design practice, it is usually desired to know the factor of safety on the ultimate loadings. For this reason ultimate load conditions have been considered. In applying the theory it is necessary to consider a limiting value of one reinforcement stress, or of the concrete stress. Then the associated stresses which are consistent with ultimate loadings can be computed.

It has been shown that wherever it is desired to reinforce for the principal tensions of a stress field, reinforcement must be orientated precisely in the directions of principal tension or else the orientation of compression in the concrete will not coincide with the direction of applied principal compressive stress. One exception occurs when equal quantities of reinforcement, placed orthogonally at 45 deg to the principal stress, are used as reinforcement for pure shear. However, in this case the concrete stress is twice the principal compressive stress.

Particular attention has been given to the problem of reinforcement for combined shear and direct stress when one of two sets of reinforcement is placed in the direction of the direct stress component. This condition is closely related to that in simple concrete beams.

Eq. (28) can be used for estimating the minimum quantity of stirrups for beams. It has been plotted in Fig. 3 for $\phi = 0$ and $\phi = 45$ deg. It is of interest to note that a concrete beam with $\phi = 0$ has a maximum theoretical shear strength only half the maximum theoretical shear strength for a beam having $\phi = 45$ deg,

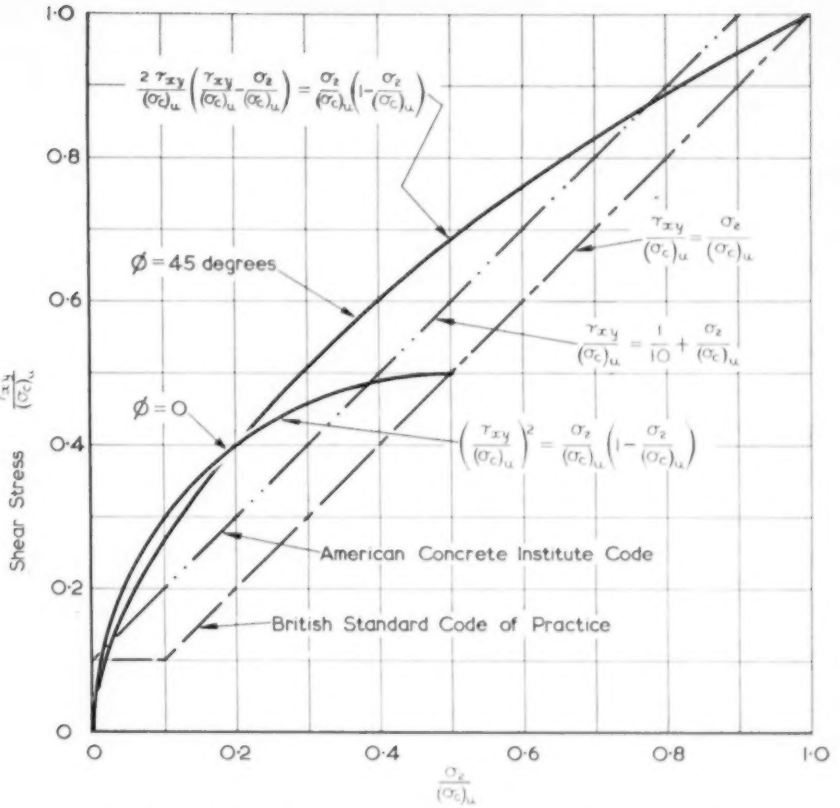

Fig. 3—Curves of minimum stirrup stress for $\phi = 0$ and $\phi = 45$ deg

yet in the low range of normal working stresses the influences of both types of reinforcement are closely related.

It must be realized, when Eq. (28) is applied to conventional beams, that ideal bonding of longitudinal reinforcement and ideal anchorage of stirrups has been presumed. It is quite possible that in tests, incipient bond and anchorage failure under high loads, in all but carefully detailed beams, may prevent the full strengths, which have been indicated possible, from being realized. In any event the influence of stirrup spacing and consequent stress concentrations at anchorages must be considerable. This is an aspect which is best left for experimental investigation.

The 1951 ACI Building Code[5] limits the shear stress of beams with only vertical stirrups to 0.08 $(\sigma_c)_u$ and with inclined (or combinations of) stirrups to 0.12 $(\sigma_c)_u$. Presuming a safety factor of 3, the ultimate shear stresses envisaged should be 0.24 $(\sigma_c)_u$ and 0.36 $(\sigma_c)_u$, respectively. It is seen from Fig. 3 that the normal range of working stresses is based on only the first part of the curves.

For comparison, the design requirements for stirrups of both the ACI Code and

the British Standard Code of Practice (BSCP) have been plotted. It will be seen that for $\sigma_z/(\sigma_c)_u = 0.1$, ultimate shear strengths three times the BSCP ultimate and 1.5 times the ACI ultimate are possible. This order of magnitude of the increased strengths has been recorded in numerous experiments.[1,3,4]

The present theoretical result and the previous confirmatory experiments need not necessarily suggest that the codes are far too conservative. The high shear strengths can only be realized when the primary reinforcement is adequate and bond strengths are good.

For instance, if $\sigma_z/(\sigma_c)_u = 0.1$ and $\tau_{xy}/(\sigma_c)_u = 0.3$ then from Eq. (24)

$$\sigma_1 - \sigma_x = \sigma_c - \sigma_z = 0.9\ (\sigma_c)_u$$

This is high. If $(\sigma_c)_u = 3000$ psi, the limiting value of $\sigma_1 - \sigma_x$ must be greater than 2700 psi; σ_z would be 300 psi and $(\tau_{xy})_{ultimate}$ would be 900 psi.

In beams there are locations where the quantity of primary reinforcement is superfluous, and so the opportunity is often available to use a minimum of stirrup reinforcement. It may also ease construction costs to use fewer stirrups and more primary steel.

In general, however, if the amount of stirrup reinforcement is decreased, only half the amount of longitudinal steel seemingly required for compensation need be placed. This is because one flange can remain in compression, and only the tension flange will require additional steel. Using this criterion, it can be shown then that the greatest economy in use of vertical stirrups may be made when the stirrups are proportioned to carry a stress of $(1/\sqrt{2})\ (S/bjd)$, and tensile flange steel is provided to carry a load of $M/jd + S/\sqrt{2}$. Similar results could be computed for other values of ϕ.

CONCLUSIONS

It is considered that the present theory gives a rational method of determining quantities of tensile reinforcement for concrete under two-dimensional stress, and can be particularly helpful for proportioning shear reinforcement in highly stressed beams and walls.

The assumption of some codes that concrete can carry a limiting tension after cracking has not been made, yet the experimental results which led the compilers of the codes to adopt that assumption (to provide an allowance for increased strength) have been justified theoretically.

Furthermore, an explanation has been given for the observed high shear strengths of lightly stirruped concrete beams, and it has been shown that these high shear strengths may only be realized when the main reinforcement is adequate to carry the sum of the primary load and induced loads due to shear. This latter aspect is frequently ignored in practice, often, it is believed, at the penalty of an encroachment upon presumed factors of safety.

ACKNOWLEDGMENT

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APPENDIX—APPLICATION TO SHEAR STRENGTHS OF SHALLOW BEAMS

Since the present paper was submitted for publication the four-part series on shear strengths of reinforced concrete beams has been published in the JOURNAL.* As those papers contain a wealth of new information and discussion on the shear strength of beams, the opportunity has been afforded the author to give a comparison of his theoretical predictions of shear strength with strengths found experimentally by Elstner, Moody, Viest, and Hognestad.

APPLICABILITY TO SHALLOW BEAMS

The analysis of the paper was developed by considering a reference element of composite concrete and reinforcement under the action of a uniform stress field. It is believed that the application remains justified in a varying stress field, provided the spacing of reinforcement is effectively close, compared with the rates of change of stress, and provided the reinforcement is able to transfer changes of load by bond tractions without slippage at any point. However, in a "shallow" or conventional reinforced concrete beam, where the variation of stress is rapid with depth and stirrups generally need mechanical anchorage, it is problematical how just or unjust application of the theory may be. Nevertheless, one may postulate an "ideally stirruped beam," for which effects of stirrup spacing, size, and shape may be neglected, and then obtain from the theory shear strengths which should represent the maximum it is possible to obtain.

Notation

f_c' = strength of concrete
r = percentage of web reinforcement
p = percentage of longitudinal reinforcement at top or bottom of beam

*Moody, K. G., Viest, I. M., Elstner, R. C., and Hognestad, E., "Shear Strength of Reinforced Concrete Beams," ACI JOURNAL, *Proc.* V. 51: Part 1, Dec. 1954, pp. 317-332; Part 2, Jan. 1955, pp. 417-436; Part 3, Feb. 1955, pp. 525-540; Part 4, Mar. 1955, pp. 697-732.

f_y = yield stress of reinforcement
b = width of beam
d = effective depth of beam
d' = distance between centers of top and bottom reinforcement
jd = effective lever arm between centers of action of longitudinal compression and tension
$V = S$ = applied shear in shear span
$v = V/bjd$ = shear stress on a cross section
v_u = ultimate value of v
A_s, A_s' = areas of longitudinal reinforcement
F = defined in Eq. (37)
F_c, F_s = defined in Eq. (38) and (39)

THEORETICAL MAXIMUM SHEAR STRENGTHS

Eq. (28) can be used for estimating the minimum quantity of stirrups for beams. It has been plotted in Fig. 3 for both vertical stirrups ($\phi = 0$) and for stirrups inclined at 45 deg ($\phi = 45$). For this minimum quantity of stirrups to be fully effective the longitudinal reinforcement must be adequate to meet the requirements of Eq. (29), as is explained later.

For aid in comparison with the conventional notation for beams

$$\frac{\sigma_2}{(\sigma_c)_u} \equiv \frac{rf_y}{f_c'} \quad \text{where } (\sigma_c)_u = f_c', \text{ and } \sigma_2 = rf_y \qquad (34)$$

and

$$\frac{\tau_{xy}}{(\sigma_c)_u} \equiv \frac{v}{f_c'} \quad \text{where } v = \frac{V}{bjd} \qquad (35)$$

Given the ultimate stress of the concrete f_c', the percentage of stirruping r, and the yield stress of the stirrups f_y, the theoretical maximum shear strengths $\tau_{xy}/(\sigma_c)_u$ may be computed, or read from the graphs.

This has been done for the 35 restrained beams with web reinforcement described in Part 3* of the series (see Table 3, p. 529), and comparisons of computed and experimental maximum shear strengths are given in Table A1.

SHEAR STRENGTHS IN THE PRESENCE OF FLEXURE

It follows from the nature of Eq. (28) and (29) that the theoretical maximum shear strength of a beam with given stirruping is not diminished by flexure, provided the strength of the longitudinal reinforcement is more than a certain minimum, which is dependent on both the bending moment and the shear. For an amply stirruped beam, failure would theoretically occur when the longitudinal reinforcement, either tensile or compressive, is inadequate for its function.

It is beyond the scope of this paper to establish criteria for failure by crushing of the longitudinal compressive zone of a beam. However, requirements for adequacy of tensile reinforcement can be established.

Eq. (24) may be rewritten

$$\sigma_1 = \sigma_s + \sigma_c - \sigma_3 \qquad (24a)$$

At a section of contraflexure of a beam, for instance at $x = 0$ of Fig. A1, it may be assumed that, either

$\sigma_s = 0$, or $b \int \sigma_s \, dy = 0$

*Elstner, R. C., Moody, K. G., Viest, I. M., and Hognestad, E., "Shear Strength of Reinforced Concrete Beams. Part 3—Tests of Restrained Beams with Web Reinforcement," ACI JOURNAL, Feb. 1955, *Proc.* V. 51, pp. 525-540.

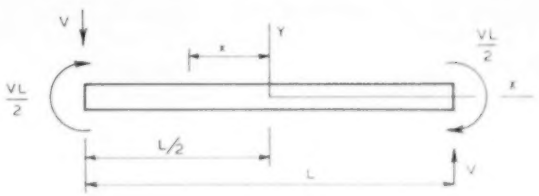

Fig. A1—Beam dimensions and forces

consequently

$$\sigma_1\, bd' = (\sigma_c - \sigma_2)\, bd' \quad \ldots\ldots\ldots\

TABLE A1—COMPARISON OF ILLINOIS* EXPERIMENTAL STRENGTHS WITH THEORETICAL MAXIMUM SHEAR STRENGTHS FOR GIVEN WEB REINFORCEMENT

Beam No.	Stirrups r, percent	Stirrups yield stress f_y, ksi	$\sigma_2 = rf_y$, psi	Concrete strength $(\sigma_c)_u = f_c'$, psi	$\frac{\sigma_2}{(\sigma_c)_u}$	Theoretical strength $\frac{\tau_{xy}}{(\sigma_c)_u}$	Test strength $\frac{v_u}{f_c'}$	Ratio $\frac{v_u}{\tau_{xy}}$
I 10a	0.52	47.3	246	3070	0.080	0.272	0.246	0.90
10b	0.52	47.3	246	2810	0.088	0.282	0.228	0.81
I 11a	0.95	44.0	418	3560	0.117	0.321	0.249	0.78
11b	0.95	44.0	418	3180	0.131	0.337	0.253	0.75
I 12a	1.47	41.2	606	4000	0.151	0.358	0.220	0.61
12b	1.47	41.2	606	3220	0.183	0.392	0.229	0.58
I s	1.72	47.6	819	3470	0.236	0.424	0.294	0.69
I t	2.14	47.6	1019	3700	0.275	0.446	0.300	0.67
I 13a	0.74	47.3	350	3460	0.101	0.257	0.248	0.96
13b	0.74	47.3	350	2860	0.122	0.300	0.276	0.92
I 14a	1.35	44.0	594	3510	0.169	0.363	0.330	0.91
14b	1.35	44.0	594	3600	0.165	0.357	0.309	0.87
I 15a	2.09	41.2	861	3330	0.259	0.465	0.341	0.60
15b	2.09	41.2	861	3710	0.232	0.436	0.379	0.87
I 16a	1.47	41.2	606	3610	0.168	0.374	0.348	0.93
16b	1.47	41.2	606	3240	0.187	0.390	0.403	1.03
I u	0.52	53.8	280	3740	0.075	0.263	0.198	0.75
I v	0.70	53.8	377	3580	0.105	0.306	0.220	0.72
I w	0.95	45.8	435	4210	0.103	0.304	0.197	0.65
I x	1.47	47.6	700	3830	0.183	0.386	0.262	0.68
I y	0.52	53.8	280	4790	0.059	0.235	0.213	0.91
I z	0.70	53.8	377	4850	0.078	0.267	0.212	0.79
I α	0.95	45.8	435	5070	0.086	0.280	0.238	0.85
I β	1.47	47.6	700	5130	0.136	0.342	0.252	0.74
II 21a	0.52	47.3	246	3580	0.069	0.253	0.228	0.90
21b	0.52	47.3	246	3640	0.068	0.251	0.204	0.81
II 22a	0.95	44.0	418	3000	0.139	0.346	0.262	0.76
22b	0.95	44.0	418	2710	0.154	0.361	0.281	0.78
II 23a	1.47	41.2	606	3230	0.188	0.390	0.244	0.63
24b	1.47	41.2	606	3160	0.192	0.394	0.291	0.74
II e	1.72	43.5	748	3420	0.219	0.413	0.300	0.73
II f	2.14	43.5	931	3330	0.280	0.449	0.268	0.60
IV m	0.52	53.8	280	2860	0.098	0.297	0.202	0.68
IV n	0.95	45.8	435	3710	0.117	0.321	0.212	0.66
IV o	1.47	47.6	700	2420	0.205	0.402	0.253	0.63

*Op. cit., Table 3, p. 529.

If $M = Vx$ is the moment at a section at x, the compressive flange force is

$$F_c = \frac{M}{jd} - F = \frac{M}{jd} - \tfrac{1}{2}\,\sigma_1\,bjd \qquad (38)$$

and the force in the tensile reinforcement is

$$F_s = \frac{M}{jd} + F = \frac{M}{jd} + \tfrac{1}{2}\,\sigma_1\,bjd \qquad (39)$$

Provided then, that a value of jd may be assumed, the quantity of tensile reinforcement required may be computed from Eq. (39).

As mentioned earlier, the minimum value of $\sigma_2 = rf_y$ may be found from Eq. (28). However, whatever the value of σ_2,

$$(\sigma_1)_u = \beta\,\sigma_2 \qquad (40)$$

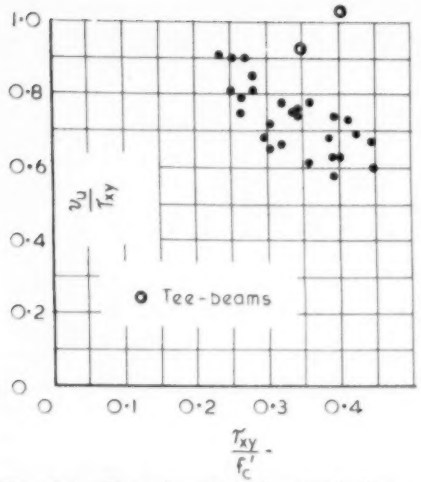

Fig. A4—Plot of ratios of actual shear strengths to theoretical maximum shear strengths for vertical stirrups

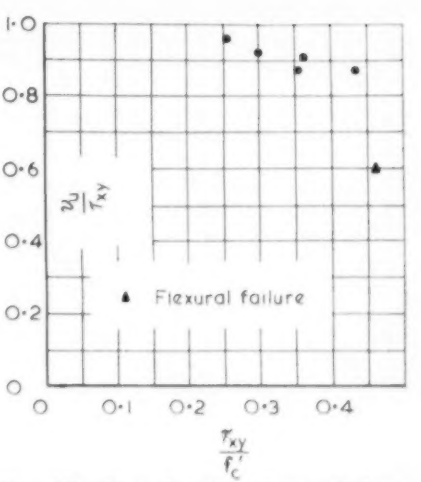

Fig. A5—Plot of ratios of actual shear strengths to theoretical maximum shear strengths for inclined stirrups

where β may be found from Eq. (24), (27), and (32) and is, for

$$\phi = 0, \quad \beta = \left(\frac{\tau_{xy}}{\sigma_z} \right)^2 \text{ i.e., } \tau_{xy} = \sqrt{\sigma_z(\sigma_1)_u}$$

and for

$$\phi = 45 \text{ deg}, \quad \beta = \frac{2\tau_{xy}\,(\tau_{xy} - \sigma_z)}{\sigma_z^2}$$

For instance, for vertical stirrups it follows that if the stirrups are proportioned to carry a stress of

$$\sigma_z = \frac{1}{\sqrt{\beta}} \frac{V}{bjd} \qquad (41)$$

the "tension flange" reinforcement must carry a force of

$$F_s = \frac{M}{jd} + F, \text{ where } F = \tfrac{1}{2}(\sigma_1)_u\, bjd = \tfrac{1}{2} V\sqrt{\beta} \qquad (42)$$

For a shear span of a given beam (presuming an adequate compressive flange strength) the ultimate loading can be computed, given the ultimate strength of the concrete f_c', the percentage of stirruping r, the percentage of tensile reinforcement p, and the reinforcement yield stresses f_y. About three cycles of successive approximation suffice for a close estimate, where a trial value of the ultimate load P is averaged with the associated value of P obtained from the computed shear stress.

Such computations have been made for 20 of the restrained beams with vertical stirrups described by Elstner, Moody, Viest, and Hognestad, and the comparisons are given in Table A2.

DISCUSSION

The ratios of the actual shear strengths to the theoretical maximum shear strengths given in Table A1 have been plotted in Fig. A4 and A5. Fig. A4 shows the ratios for vertical stirrups and Fig. A5 for stirrups inclined at 45 deg.

TABLE A2—COMPARISON OF ILLINOIS EXPERIMENTAL STRENGTHS WITH THEORETICAL STRENGTHS FROM PRESENT THEORY

Beam No.	Ultimate shear in test $\frac{v_u}{f_c'}$	$f_c' = (\sigma_c)_u$, psi	$\sigma_2 = rf_y$, psi	$\sigma_1 = 2pf_y$, psi	For section at support					Section $d/2$ from support				
					Calc. P_u, kips	$(\sigma_1)_u$, psi	$\sqrt{\sigma_2(\sigma_1)_u}$, psi	$\frac{\tau_{xy}}{(\sigma_c)_u}$	$\frac{v_u}{\tau_{xy}}$	Calc. P_u, kips	$(\sigma_1)_u$, psi	$\sqrt{\sigma_2(\sigma_1)_u}$, psi	$\frac{\tau_{xy}}{(\sigma_c)_u}$	$\frac{v_u}{\tau_{xy}}$
I 10a	0.246	3070	246	4350	158	2125	723	0.236	1.04	180	2770	823	0.268	0.92
10b	0.228	2810	246	4350	158	2125	723	0.257	0.89	180	2770	823	0.293	0.78
I 11a	0.249	3560	418	4350	186	1733	851	0.239	1.04	219	2430	1003	0.282	0.88
11b	0.253	3180	418	4350	186	1733	851	0.268	0.94	219	2430	1003	0.316	0.80
I 12a	0.220	4000	606	4350	205	1457	940	0.235	0.94	249	2165	1141	0.285	0.77
12b	0.229	3220	606	4350	205	1457	940	0.292	0.78	249	2165	1141	0.354	0.65
I s	0.294	3470	819	4117	211	1144	967	0.279	1.05	264	1800	1210	0.348	0.85
t	0.300	3700	1019	4117	221	999	1010	0.273	1.10	282	1640	1291	0.349	0.86
I u	0.198	4790	280	2422	109	881	496	0.133	1.59	131	1270	598	0.160	1.24
v	0.220	4850	377	2422	117	769	538	0.150	1.47	144	1150	660	0.184	1.19
w	0.197	5070	435	2422	121	713	556	0.132	1.49	151	1090	691	0.164	1.20
x	0.262	5130	700	2422	134	477	612	0.160	1.64	172	900	788	0.206	1.27
II 21a	0.228	3580	246	2486	228	1445	595	0.166	1.37	254	1760	662	0.185	1.23
21b	0.204	3640	246	2486	228	1445	595	0.163	1.25	254	1760	662	0.182	1.12
II 22a	0.262	3000	418	2486	275	1230	717	0.239	1.10	312	1595	814	0.272	0.96
22b	0.281	2710	418	2486	275	1230	717	0.265	1.06	312	1595	814	0.300	0.94
II 23a	0.244	3230	606	2486	309	1075	807	0.250	0.98	357	1465	931	0.289	0.84
23b	0.291	3160	606	2486	309	1075	807	0.255	1.14	357	1465	931	0.309	0.94
II e	0.300	3420	748	2353	316	909	825	0.241	1.24	375	1280	977	0.286	1.05
f	0.268	3330	931	2353	336	822	875	0.263	1.02	404	1200	1053	0.316	0.85

Close agreement is obtained for the beams with inclined stirrups. It is noted that all these but one had high quantities of longitudinal reinforcement, and for the shear spans of the tests, flexure should have little effect on the magnitude of the maximum shear.

For the beams with vertical stirrups there is close agreement for the smaller quantities of stirrups, and except for the two T-beams tested, there is a consistent lowering of the ratio v_u/τ_{xy} for larger quantities of stirrups.

This lowering of v_u/τ_{xy}, shown in Fig. A4, is partly explained by the computations of Table A2, which indicate that, for the sections of high bending moment, the quantity of longitudinal reinforcement, though generous for pure flexure, is not adequate for marshalling the full theoretical ultimate shear strengths. However, the same argument should be applicable to the two T-beams, but these curiously do not show the reductions of actual shear strength noted for the rectangular beams with vertical stirrups.

Table A2 lists ratios of test to theoretical strengths computed for sections at the supports and for sections a distance from the supports equal to half the effective depth of the beams. This latter listing is considered to be the more rational one in the present comparison of theoretical and experimental strengths. It shows a reasonable and encouraging correlation.

It is the author's opinion that the high local stresses from point loadings in the conventional laboratory planar testing systems must have an appreciable influence on behavior. Of great interest would be, say, a spatial test where a continuous primary beam is alternately loaded upward and downward by transverse secondary beams of identical sections. Such a test would eliminate from the primary beam the undesirable influence of point loads, as well as the likely tendency in most laboratory tests for "local arching" within the beam, between loads applied alternately at the upper and lower surfaces.

For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Title No. 53-16

Some Effects of Carbon Dioxide on Mortars and Concrete*

By I. LEBER† and F. A. BLAKEY‡

SYNOPSIS

Tests have been made to determine the effect of gaseous carbon dioxide on the strength and shrinkage of mortars and concretes. Results from work previously published appeared somewhat contradictory, but it now seems that strength is increased and shrinkage decreased if specimens undergo a period of curing in carbon dioxide immediately after demolding.

On the other hand, an immediate sharp increase in shrinkage is recorded if specimens are allowed to dry for several days before carbon dioxide treatment is started. The contraction does not continue with time, but appears to be stabilized.

The influence of carbon dioxide under pressure, and of storage in air free from carbon dioxide were studied briefly. Some consideration has been given to the way in which the gas is bound by the cement.

INTRODUCTION

It has long been accepted that atmospheric carbon dioxide has an effect on concrete, but information on the nature and extent of this effect is meager.

From routine shrinkage measurements on concretes and mortars made at this Division it appeared that carbon dioxide might have an important influence on this shrinkage, and a special study of this possibility was begun. An examination of the literature on shrinkage was not particularly helpful, as the results from investigations dealing with the influence of gaseous carbon dioxide were contradictory. There was no indication as to what factors associated with carbon dioxide might or might not be important, and this made it impossible to draw up a definite plan for the work. The curing treatments used on successive mixes were therefore chosen as the work progressed. In retrospect it seems that some of the choices were bad, and results from some of the curing procedures yielded nothing new, but at the time it was impossible to forecast what treatments might be useful.

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REVIEW OF PREVIOUS INVESTIGATIONS

Brady[1] calculated from the densities of crystalline calcium hydroxide and calcite that the effect of carbonation in a 1:3 cement/sand mortar should be an increase in volume of about 0.05 percent as the calcium hydroxide is converted to calcium carbonate. However, an experimental study showed an increase in shrinkage after carbon dioxide treatment. The specimens were left for 7 days in moist air and then partly dried out in an atmosphere of 63.5 percent relative humidity and 20 C for 3 days. Some were then kept in carbon dioxide and others in air at the same humidity and temperature. The specimens stored in air showed only a slight shrinkage, whereas specimens stored in carbon dioxide showed a further shrinkage of the order of two-thirds of the initial shrinkage. Schwiete[2] found that various cements showed from 13 to 33 percent greater shrinkage when their mortars were cured for 180 days in carbon dioxide-rich atmosphere than when they were cured in air. Yoshida[3] discussed the carbonation as it proceeds under different conditions, and considered that increased attention to the role of carbon dioxide in volume change is justified.

Lea and Desch[4] stated that the rate of absorption of carbon dioxide by cement products is slow in materials saturated with water and increases with decreasing humidity. In a water-saturated atmosphere a skin of calcium carbonate is formed on the wet surface (Bessey[5]), preventing penetration of the carbon dioxide. Carbonation of concrete, both in the field and in test specimens, is therefore to be expected more under dry than under wet conditions. The time when the carbonation takes place seems to be important. Mansfield[6] claimed that carbonation is detrimental when it occurs before the portland cement hydration compounds are formed, but is beneficial when it follows hydration.

Meyers[7] investigated the effect of carbon dioxide on the strength and shrinkage of mortars composed of one part of cement to four parts of sand. He cured all specimens in water for 7 days and then stored them in sealed containers. A carbon dioxide absorbent was placed in the containers with those specimens to be used as controls. The pore water was removed by vacuum from the remaining specimens and they were then kept in carbon dioxide for 10 days at 50 psi. Some of these specimens were then further treated for 15 days and others for a month with carbon dioxide at 150 psi. It was shown that shrinkage of the specimens treated with carbon dioxide at 150 psi for one month was only 30 to 40 percent of that for untreated specimens, and that they attained considerably higher strength.

CARBONATION OF HYDRATED MORTARS

The first tests were made on mortars, each mix composed of one part by weight of normal portland cement to four parts by weight of quartz sand, with a water-cement ratio of 0.6. One brand of cement conforming to Australian Standard A2 and one batch of sand were used throughout.

The mortars were kept in steel molds for 24 hr. Compressive strength spec-

imens were cast in 2.778- and 1-in. cubes. Shrinkage specimens were bars 1 in. square in cross section and 11.25 in. long, made in molds described in ASTM C 157-53T. For comparison some shrinkage measurements were made on bars of 2 in. cross section and 11.25 in. long. Bars of both sizes were used for modulus of rupture tests under third-point loading.

Strength tests

Tables 1 and 2 compare the effect of carbon dioxide storage at different stages on compressive strength and on extent of carbonation. Each strength figure

TABLE 1—CURING PROCEDURES AND STRENGTHS OF 1:4 (CEMENT/SAND) MORTARS

Fog room, 21 C, 99 percent R.H., days	CO_2 (moist), 21 C, 99 percent R.H., days	Constant air conditions, days	CO_2, 21 C, 65 percent R.H., days	Compressive strength at 28 days, psi	Carbonated CaO, percent§
5	—	1	21	6000	48.3
5	—	1	21	6550	46.0
6	—	2	19	5920	48.1
5	—	2	20	5920	46.5
6	—	1	20	4590	—*
—	5	1	21	4050	56.5
—	5	1	21	5120	52.8
—	6	2	19	4420	44.3
—	5	2	20	3810	46.0
—	7	—	19	4950†	46.0
—	8	—	20	4650‡	47.5
—	6	21	—	3830	—*
27	—	—	—	3070	5.5
27	—	—	—	3000	4.8
27	—	—	—	2770	4.0
6	—	21	—	2630	—*
—	—	27	—	1878	—*

*Not analyzed.
†At 27 days.
‡At 29 days.
§Calculated as percentage of original weight of cement.

represents an average result of tests on three 1-in. cubes. To find the extent of carbonation, samples were taken from the center part of a crushed 1-in. cube and from the surface layer, and analyzed for carbon dioxide. In each case it appeared that the center part was carbonated almost as much as the surface layer.

TABLE 2—CURING PROCEDURES AND STRENGTHS OF MORTARS AFTER 8 DAYS

Fog room, 21 C, 99% R.H., days	CO_2 (moist), 21 C, 99% R.H., days	Constant air conditions, days	Compressive strength, psi
—	6	1	2945
—	6	1	3175
6	—	1	1592
6	—	1	1770

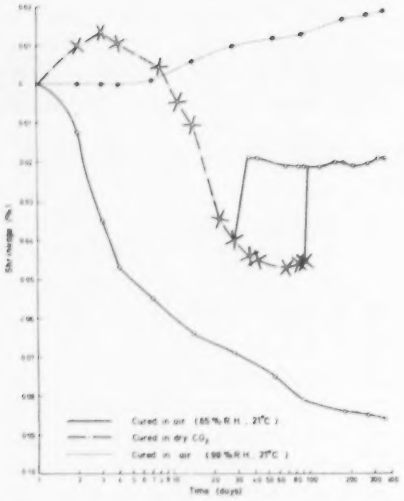

Fig. 1a—Shrinkage of mortars cured in dry carbon dioxide

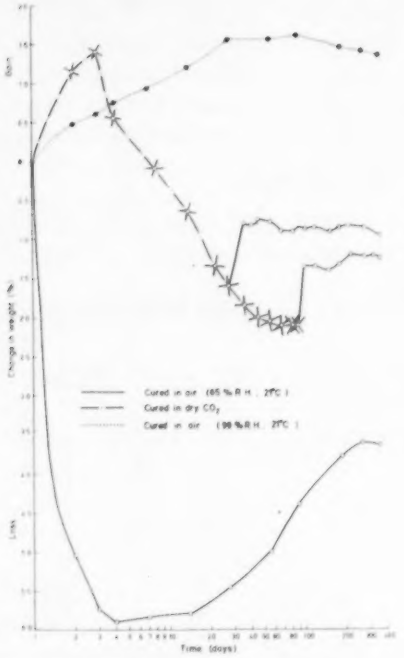

Fig. 1b—Change in weight of mortars for which shrinkage is shown in Fig. 1a

Shrinkage tests

The carbon dioxide curing equipment was the same as that used for the specimens in the strength tests, and specimens were temporarily removed from the curing for measurement when required. Some shrinkage specimens cured under various conditions were broken at different ages and analyzed for carbon dioxide. Table 3 gives the results of these analyses. Fig. 1 (a and b) and 2 (a and b) show the shrinkage and change of weight of specimens cured under various conditions.

TABLE 3—EFFECT OF CARBONATION ON 1:4 (CEMENT/SAND) MORTARS

CO_2 (dry), 21 C, days	Constant air conditions, days	CO_2 21 C, 65 percent R.H., days	Constant air conditions, days	Carbonated CaO,* percent	Fig. No.
28	63	—	—	31.9	1a and 1b
90	—	—	—	56.8	1a and 1b
—	28	63	30	29.6	2a and 2b
—	90	—	—	25.2	—

*Calculated as percentage of original weight of cement.

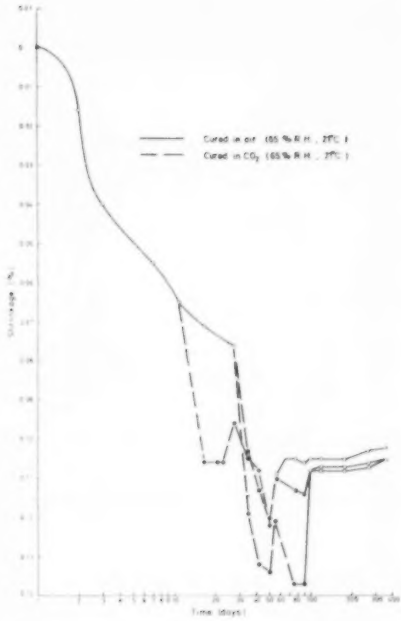

Fig. 2a—Shrinkage of mortars which have been kept in carbon dioxide after various air curing

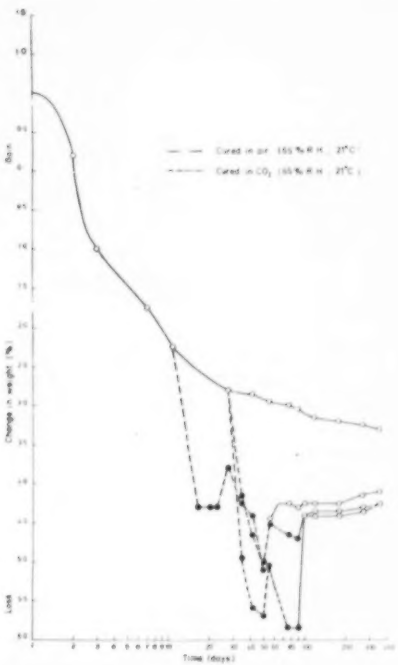

Fig. 2b—Change in weight of mortars for which shrinkage is shown in Fig. 2a

To obtain a clearer indication of the effect of carbon dioxide some specimens were stored in carbon dioxide-free air. The shrinkages and change in weight in these specimens are shown in Fig. 3a and 3b, respectively. Some of these specimens were tested for strength and analyzed for carbon dioxide at different ages; Table 4 shows the results of these tests.

CARBONATION OF CONCRETES

Further tests were made on concretes of 1:1½:2½ proportions by weight (cement:sand:stone) with a water-cement ratio by weight of 0.4. The aggregate-cement ratio of 1:4 was chosen so that results would be comparable with those from the mortars already described. The use of such a rich mix also makes the study more applicable to the type of concrete used in prestressing work, where shrinkage is an important factor that must be allowed for in design. One brand of cement (the same as that used in the mortars) and one batch of quartz sand and basalt coarse aggregate were used throughout. Cement, sand, and basalt were stored in constant air conditions before using; the mixing, molding, and air treatments were also carried out in constant air conditions.

All the mixing was done in a Cumflow mixer. Concrete specimens were

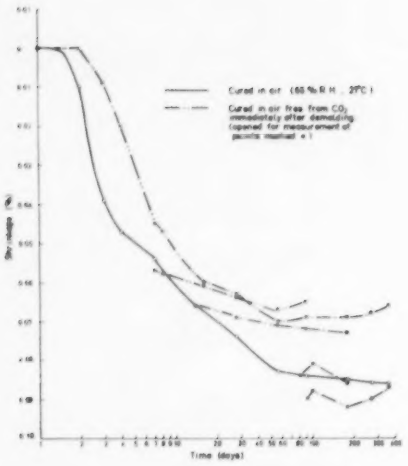
Cured in air (65 % R.H., 21°C.)
Cured in air free from CO2 immediately after demolding (opened for measurement of points marked ×)
Time (days)

Fig. 3a—Shrinkage of mortars cured in air free from carbon dioxide, compared with those cured in ordinary air

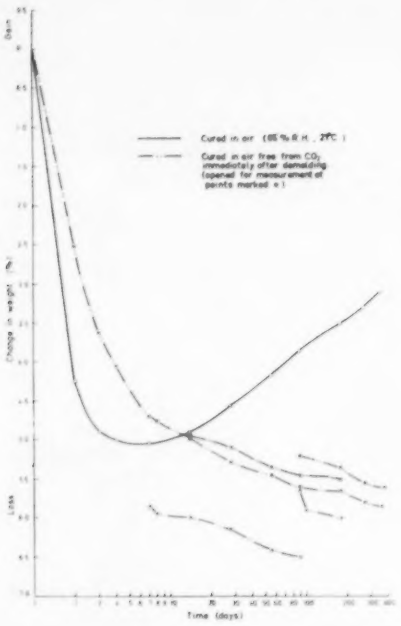
Cured in air (65 % R.H., 21°C.)
Cured in air free from CO2 immediately after demolding (opened for measurement of points marked ×)
Time (days)

Fig. 3b—Change in weight of the specimens cured in air free from carbon dioxide

TABLE 4—COMPARISON OF SPECIMENS CURED IN CO_2-FREE AIR AND IN AIR

Curing	Age of specimen at time of test, days	Compressive strength, psi	Modulus of rupture, psi	Carbonated CaO,* percent
Carbon dioxide-free atmosphere at 21 C and 65 percent relative humidity	91	1033	194	1.96†
	180	—	284	3.65‡
	180	—	288	0.95§
Constant air conditions	91	1878	410	25.0
	360	—	644	30.5

*Calculated as percentage of original weight of cement.
†Removed from curing for measurement at 7, 8, 14, 22, 28, 42, 56, 91 days.
‡Removed from curing for measurement at 14, 22, 28, 42, 56, 91, 135, 180 days.
§Removed from curing for measurement at 91, 135, 180 days.

TABLE 5—MODULUS OF RUPTURE OF CONCRETES AFTER CARBON DIOXIDE CURING

Constant air conditions, days	CO_2, 21 C, 65 percent R.H., days	Modulus of rupture, psi
—	27	1023
—	27	1200
27	—	663
27	—	618

kept in steel molds for 24 hrs. Compressive strength specimens were 2.778-in. cubes, those for tensile strength were cylinders 3 in. in diameter and 6 in. long. Shrinkage specimens were 11.25 in. long and mostly of 2 in. square cross section, but some were of 1 in. square cross section.

Carbon dioxide curing under normal pressure

The carbon dioxide curing of concrete test specimens under normal pressure was carried out in sealed steel chambers in which constant air conditions were maintained. Conditions in the fog room were the same as those described in the mortar tests, *i.e.*, 21 C and 99 percent relative humidity.

Table 5 shows the modulus of rupture of specimens cured either in carbon dioxide or in air, and Table 6 shows the compressive strength of individual specimens after various storage conditions including carbon dioxide. Similarly, Table 7 shows the tensile strengths as determined by the Brazilian test (Blakey[8]) on the cylindrical specimens.

TABLE 6—COMPRESSIVE STRENGTH OF CONCRETES AFTER DIFFERENT CURING TREATMENTS

Fog room, 21 C, 99 percent R.H., days	CO_2, 21 C, 65 percent R.H., days	Constant air conditions, days	CO_2, 21 C, 65 percent R.H., days	Compressive strength, psi
7	20	—	—	12,010
—	27	—	—	12,263
—	7	20	—	11,030
—	—	7	20	14,500
—	—	7	20	16,100
27	—	—	—	9,706
—	—	27	—	6,640
17	10	—	—	11,930
—	27	—	—	13,490
—	27	—	—	9,180
—	7	20	—	9,860
—	14	13	—	10,150
—	—	7	20	15,350
—	—	7	20	10,810
—	—	14	13	10,760
—	27	—	—	11,500
27	—	—	—	10,360
—	—	10	17	11,130

TABLE 7—TENSILE STRENGTH OF CONCRETES AFTER DIFFERENT CURING TREATMENTS

Fog room, 21 C, 99% R.H., days	CO_2, 21 C, 65% R.H., days	Constant air conditions, days	CO_2, 21 C, 65% R.H., days	Tensile strength, psi
18	9	—	—	770
—	—	7	20	855
—	—	14	13	856
—	27	—	—	710
27	—	—	—	505
—	—	27	—	336

Shrinkage tests—Several carbon dioxide curing treatments (applied in the same way as for strength tests) were used, in addition to ordinary air and moist curing. The shrinkage and change in weight curves are shown in Fig. 4a and 4b. After certain intervals some specimens were broken in bending and then analyzed for carbon dioxide. Results of these analyses are shown in Table 8.

TABLE 8—CURING PROCEDURES, MODULUS OF RUPTURE, AND EXTENT OF CARBONATION OF 1:4 CONCRETES

Fog room, 21 C, 99 percent R.H., days	CO_2, 21 C, 65 percent R.H., days	Constant air conditions, days	CO_2, 21 C, 65 percent R.H., days	Constant air conditions, days	Carbonated CaO*, percent	Fig. No.
7	48	34	—	—	28.45	4a and 4b
—	14	75	—	—	28.00	—
—	56	33	—	—	32.00	—
—	—	14	41	34	37.30	—
89	—	—	—	—	5.49	4a and 4b
—	—	89	—	—	24.30	4a and 4b

*Calculated as percentage of weight of original cement.

The expression of the degree of carbonation of the cement has been based again on the assumption that all the carbon dioxide taken up reacts with the lime to form calcium carbonate. It is likely that other carbonates such as those of sodium, potassium, magnesium, and iron are also formed, but the amounts of these must be small and of little importance in relation to the major components of cement, all of which contain lime.

Carbon dioxide curing under high pressure

Curing with carbon dioxide at high pressure was carried out in an autoclave in which test specimens for strength and shrinkage were stored for different times at different ages. The air was driven out through the open outlet valve by carbon dioxide pressure and the pressure of carbon dioxide increased to 60 psi. The humidity and temperature of the autoclave were not controlled. It would be reasonable to assume that they were higher than those of controlled

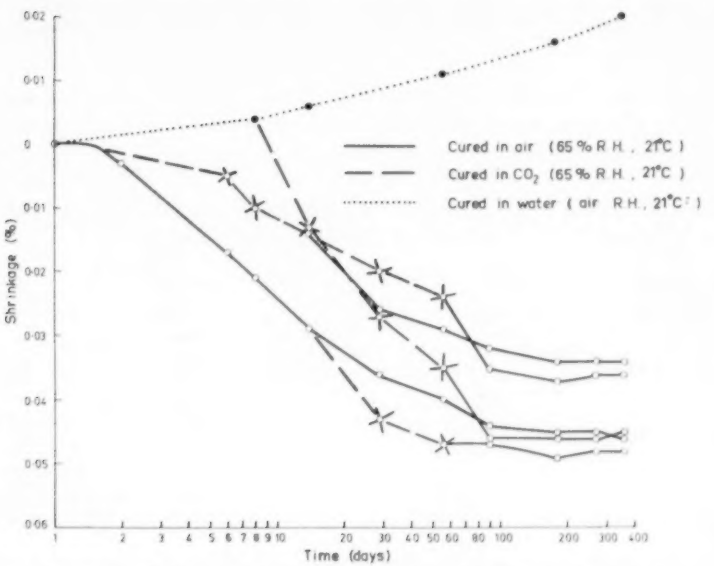

Fig. 4a—Shrinkage of concretes (2 x 2 x 11.25-in. specimens)

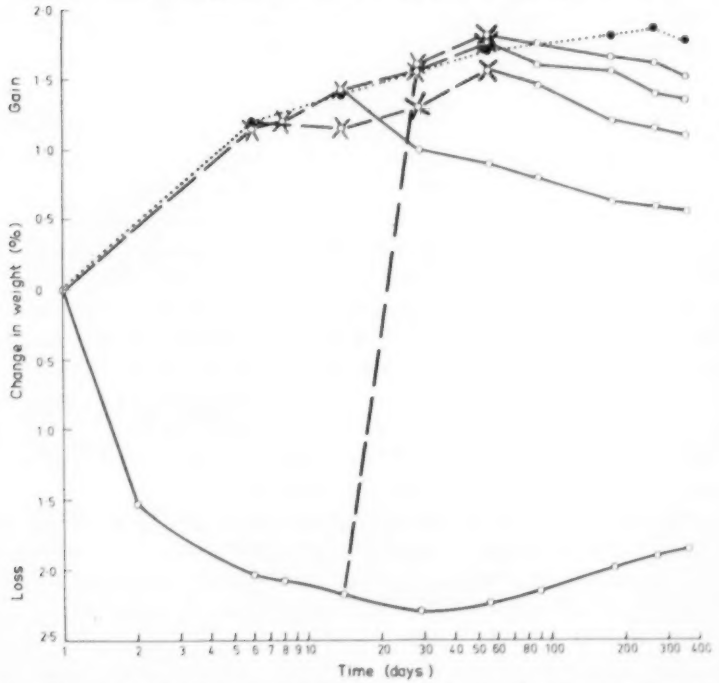

Fig. 4b—Change in weight of concretes for which shrinkage is shown in Fig. 4a

conditions because of the heat of reaction and the water liberated by the reaction.

Strength tests—The test and control specimens were tested at an age of 14 days. Table 9 shows curing procedures and compressive strength results, each figure representing a mean of the results on three specimens.

TABLE 9—STRENGTH OF CONCRETES TREATED WITH CARBON DIOXIDE UNDER HIGH PRESSURE

Fog room, 21 C, 99 percent R.H., days	Constant air conditions, days	CO_2, 60 psi, days	Constant air conditions, days	Compressive strength, psi
—	—	2	11	8440
—	7	2	4	9500
13	—	—	—	8780
—	—	1	12	7470
—	1	2	10	8230
—	3	3	7	9440
7	—	6	—	10370
13	—	—	—	9700
—	—	—	13	5860

Shrinkage tests—For shrinkage tests the curing procedures with carbon dioxide under pressure were similar to those used for strength tests. Fig. 5a shows shrinkage curves for specimens after these curing procedures; Fig. 5b shows the corresponding change in weight curves.

DISCUSSION

The tests of mortars and concretes showed increases in strength with increasing carbonation, more marked with mortars than concretes.

The amount of carbon dioxide in the mortars and concretes proves clearly that its action was not restricted to the calcium hydroxide produced during the hydration of the cement, but went further, acting on other constituents of the cement. To what extent these constituents were affected would be hard to determine, as it would not be correct to assume that there is a defined order of successive carbonation. If carbon dioxide decomposes the products of the hydration of di- and tri-calcium silicates, water and hydrous silica will be formed. This hydrous silica is insoluble in diluted hydrochloric acid (Meyers[7]), a fact which could lead to serious mistakes when determining cement or mix proportions in old and carbonated concrete by the Florentin method.

It has also been suggested that there could be an absorption of carbon dioxide by crystalline forms of cement constituents. Marshall[9] showed that chabazite can absorb carbon dioxide. Hey's theory (quoted by Marshall[9]) postulates the presence of small pores in the crystals. The size of the pores is considered to be just greater than that of a water molecule. In set cements these pores would be filled with saturated calcium hydroxide, which would react with carbon dioxide to form calcium carbonate. If there should be any excess water in the pores of

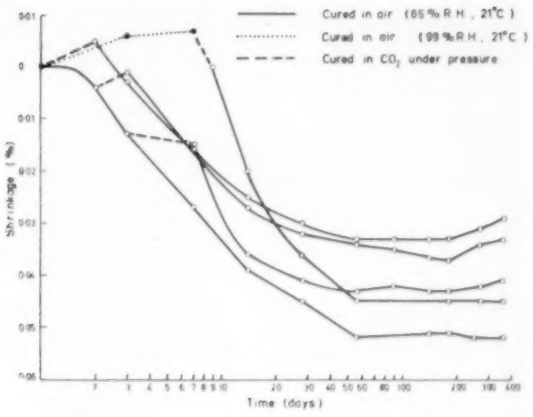

Fig. 5a—Shrinkage of concretes treated with carbon dioxide under high pressure

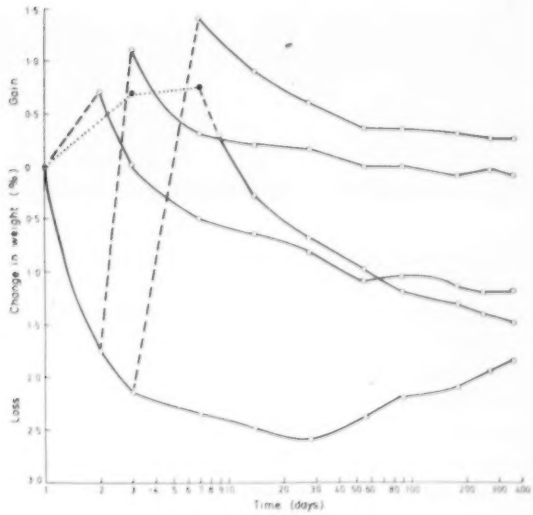

Fig. 5b—Change in weight of concretes for which shrinkage is shown in Fig 5a

the cement it is unlikely that much carbon dioxide would be dissolved, as water dissolves only about its own volume of carbon dioxide under atmospheric conditions.

X-ray studies were made on samples taken from the 1 in. square bars. The samples were ground, passed through a 200-mesh sieve to separate out as much sand as possible, and then x-rayed. Fig. 6 (1) shows the x-ray pattern of a mortar cured for 5 days in a moist atmosphere with carbon dioxide, partly dried out in air for 1 day, and then stored for 21 days in carbon dioxide (Table 1). The amount of carbonated calcium oxide, as shown by chemical analysis, was 56.5 percent. Calcium carbonate lines can be seen clearly in the x-ray pattern, and

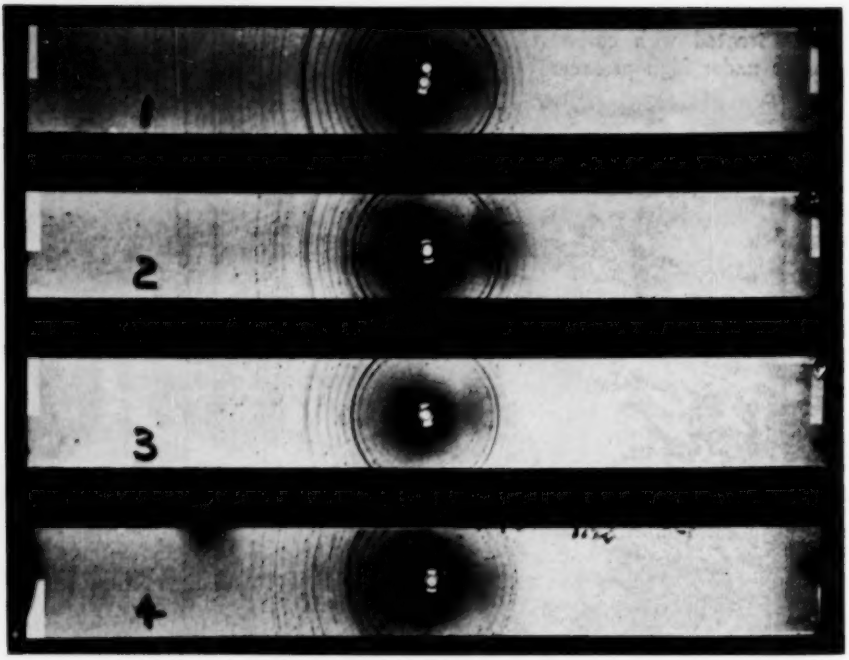

Fig. 6—X-ray patterns: (1)—Mortar cured for 5 days in moist atmosphere with CO_2, air-dried for 1 day, then stored in CO_2 for 21 days. (2)—Mortar stored in air for 540 days. (3)—Mortar cured in fog for 6 days, dried for 2 days, then cured in CO_2 for 19 days. (4)—Mortar cured in CO_2-free atmosphere for 91 days

no lines of calcium hydroxide can be seen. The calcium carbonate pattern is much weaker in Fig. 6 (2), which is for a mortar stored in air for 540 days. Here also no calcium hydroxide lines are visible. Fig. 6 (3) is for a mortar subjected to fog curing for 6 days, partly dried out for 2 days, and then cured in carbon dioxide for 19 days (Table 1). Fig. 6 (4) shows the whole x-ray pattern for a mortar cured for 91 days in a carbon dioxide-free atmosphere (Table 4). There is a strong line for crystalline calcium hydroxide, some for silica, and none for calcium carbonate. The x-ray studies, although not quantitative, show that a large amount of the carbon dioxide determined by chemical analysis is present in the form of crystalline calcium carbonate.

The specimens treated with carbon dioxide appeared to develop a slightly darker color than those cured in ordinary air conditions. The mortar cured in carbon dioxide was a yellowish color, but that cured in an ordinary atmosphere was creamy-white. The darker color of the carbonated specimen could be due to hydrous ferric oxide which is formed by the carbonation of hydration products of tetracalcium alumino-ferrite.

As already stated, curing in carbon dioxide immediately after demolding re-

duces the shrinkage of mortars. Brady[1] calculated that the carbonation of calcium hydroxide should result in a linear expansion. Further action of carbon dioxide transforms cement gel to a crystalline structure of calcium carbonate with inter-crystalline pores filled with hydrous silica, hydrous alumina, or hydrous ferric oxide. All these reactions should result in a linear expansion (Meyers[7]), but our tests on specimens which were partly dried out in air and then cured in carbon dioxide showed a greater shrinkage than in the control specimens cured in air.

It would appear that only immediate treatment with carbon dioxide results in reduced shrinkage. The hydration of cement particles proceeds for a long time, forming cement gel with fixed water, but it might be that with early carbonation of a thin specimen the hydration is slowed down. The water which would have become "bound" during hydration remains loose and is evaporated, so that less gel is formed which could add subsequently to the magnitude of the shrinkage.

However, the test results for loss in weight and especially for strength suggest that the hydration took place, and it would then appear that the water freed by the carbonation was supplied to the rest of the cement gel and aided the hydration in primary reaction. The secondary reaction of carbonation later transformed the cement gel into a crystalline calcium carbonate embedded in a binder of hydrous silica. In any case it would seem that water action has more effect than carbon dioxide, and the role of carbon dioxide in the aging processes in cement may be rather catalytic.

In the carbon dioxide treatment, where the specimens were first allowed to dry and then carbonated, a marked increase in shrinkage took place. It seems, however, that this increase stopped after a certain period and no further shrinkage took place. It also appears that this shrinkage level would be reached by ordinary air-cured specimens after a long time. The lower strength figures in this curing procedure correspond to the lesser amount of carbonation.

All the specimens were examined microscopically. Crazing, attributed by many authors to carbon dioxide, was not found. The air-cured specimens showed a few slight fissures which could not be found on carbon dioxide-cured samples.

CONCLUSIONS

1. Carbon dioxide curing considerably increases the strength of mortars and concretes at 28 days.*

2. Carbon dioxide can have either a beneficial or a detrimental effect on shrinkage of mortars and concretes.

3. The time at which the mortars and concretes are exposed to the carbon dioxide is important. The first 10 days after demolding could be considered as critical for all reactions and changes that will develop later.

4. It would appear that carbon dioxide reduces shrinkage in that it affects

*Some tests done subsequently on specimens up to 1 year old suggest that this condition may be extended to later ages.

the hydration of cement compounds. Carbon dioxide may not have a direct influence on the hydration reactions but may produce another stage of water stabilization. Loss in weight curves of carbonated specimens shows that after a long time the water content furnished by carbon dioxide action can be taken as stable or "fixed."

5. Carbon dioxide apparently not only reacts with the calcium hydroxide resulting from hydration in mortars and concrete but decomposes other constituents, forming calcium carbonate.

6. The carbon dioxide treatments at ordinary pressure appear to have been more favorable to strength and shrinkage than those at high pressures.

It is necessary to emphasize that these conclusions follow from experiments under certain limited conditions—fairly short periods of carbon dioxide curing both at ordinary and high pressures, only two types of mix, and so on. There is ample evidence that slight changes in carbon dioxide treatment, such as the age at which treatment starts, will produce completely contradictory results.

ACKNOWLEDGMENT

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For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Title No. 53-17

Analysis of Inelastic Bending Stress in Concrete Beams*

By JAMES M. PRENTIS†

SYNOPSIS

A method is given for determining the inelastic bending stresses in a concrete beam as it is tested to destruction. It is necessary only to measure the strain distribution in the concrete at a succession of load stages. The steel force is also deduced from these readings. A sample calculation illustrates the method.

INTRODUCTION

The problem of determining the bending stresses in a concrete beam at the ultimate bending moment is of both practical and academic interest. Increasing attention is being paid to ultimate strength theories which inevitably use factors dependent on the concrete stress distribution. Academic interest is aroused by the difficulty of measuring the stresses.

A recent paper by Hognestad, Hanson, and McHenry[1] summarizes various methods of obtaining the stresses by direct measurement. The common difficulty experienced is, that the introduction of a stress-measuring device into the body of a beam upsets the stresses being measured. In 1951 the author proposed a method of determining the stresses indirectly from strain measurements and applied it to results obtained from a test on an end-anchored prestressed concrete beam.[2]

As originally presented the method requires the steel force to be known. If large diameter reinforcement is used strain gages can be attached to it enabling the steel force to be deduced. In the case of end-anchored prestressed beams there is little difficulty in inserting a dynamometer between the anchorage and the end of the beam so that, provided there is little friction between the steel cable and the surrounding concrete, the steel force is again obtained easily. In either of these cases the author's method, as originally published, provides a check equation. This evaluates the concrete stress in the bottom (tension) concrete fibers of the beam. This stress is, of course, zero when the beam is cracked.

Measurement of the steel force becomes difficult when small gage, high tensile

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wire reinforcement is used in pre-tensioned or grouted post-tensioned beams. A method of determining the concrete stresses in beams of this type is given below. The check equation mentioned above is modified and used to determine the steel force. The analysis can be adapted to any shape of beam cross section, but in this paper only the rectangle is considered.

TEST PROCEDURE

Let us suppose that a pre-tensioned concrete beam using small diameter wire reinforcement is to be tested to destruction in bending. Due to creep and shrinkage the pre-tension will only be known approximately. The precise steel force can be obtained, however, because generally the concrete will not be strained inelastically until a bending moment considerably in excess of the cracking moment is applied.

Sufficient gages must be attached to the beam to enable the bending strain distribution to be plotted at a series of load stages throughout the test. Recently the author published a description of a gage which measures the strain in the top surface of the beam and the radius of curvature.[3] The two readings enable strain distribution to be deduced. It is more customary to fix a series of gage stations down the side of the beam and use a demountable gage. Care must be taken that there are always sufficient active gages above the neutral axis to give the strain distribution accurately, *i.e.*, a minimum of three gages. It is generally considered desirable to measure deflections at the same time.

Under normal conditions all the gage readings will change steadily and the deflection will increase steadily as the load is gradually increased. When the cracking load is reached there is a jump in the instrument readings, which change at a greater rate as further load is applied. Once the beam has visibly cracked the load should be removed. The rate of change of beam curvature with bending moment can be calculated from the readings so far taken. Thus Young's modulus can be deduced for the concrete and its value checked by the deflections.

On the second loading, strain and deflection readings are taken until the beam collapses. In the early stages the second test merely repeats the first with the same regular change in instrument readings; the top and bottom fiber strains and the deflection give straight line graphs when plotted against load. This continues until the bending stress in the bottom fibers just cancels the prestress there. On applying further load the previously formed cracks start to reopen, causing discontinuities in the load-strain and load-deflection graphs. These discontinuities determine the load at which the bottom fiber concrete stress is zero. Normally the stress distribution may be assumed triangular at this stage with the center of compression located one-third of the way down the beam. The distance from this point to the center of the reinforcement is the lever arm, which is divided into the external bending moment to give the steel force.

Using the previously determined Young's modulus the total concrete strains can be calculated. With this datum the concrete strains are known at all loads. By applying the analysis to follow, the steel force can be computed for all subsequent load stages together with the concrete stress.

ANALYSIS

Notation

The symbols used in this paper are defined below. Most of them are illustrated in Fig. 1.

- b = width of a rectangular beam
- d = distance from centroid of reinforcement to compression edge
- f = stress in concrete
- f_c = stress in concrete fibers at compression edge
- k = ratio giving position of neutral axis; the distance from the neutral axis to the compression edge = kd
- M = bending moment
- T = total force in reinforcement
- x = distance from neutral axis in direction of compression edge
- ε = strain in concrete at x
- ε_c = strain in concrete fibers at compression edge
- $\delta( )$ = an incremental change in the numerical value of the bracketed quantity.
- $\phi\ (\varepsilon)$ = f, states that f is some function (unspecified) of ε

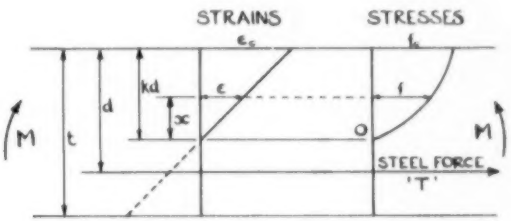

Fig. 1—Stress and strain distributions in a concrete beam

Fig. 1 shows the stress and strain distributions in a concrete beam. Assuming that the concrete carries no tensile stresses* the compressive stresses can be summed and equated to the tensile force in the steel

$$T = b\int_{x=0}^{x=kd} f\,dx$$

Assuming further that plane sections remain plane† the concrete strain is given by

$$\varepsilon = \varepsilon_c \times x/kd$$

changing the variable from x to ε we find

$$\frac{T}{bd} \times \frac{\varepsilon_c}{k} = \int_{\varepsilon=0}^{\varepsilon=\varepsilon_c} \phi(\varepsilon)\,d\varepsilon$$

where $f = \phi\ (\varepsilon)$ denotes a general but unspecified relationship between concrete stress and strain.

If it is assumed that the stress is uniquely defined by the strain (implying that the strain is independent of the stress history and that time-dependent strains

*This assumption is not essential. The effect of some tensile stresses existing below the neutral axis can be allowed for, but it is generally not significant and not worth consideration.

†Strictly, this follows automatically if four-point loading is used to give pure bending.

are negligible) this expression may be differentiated with respect to ε_c. The bracketed quantity on the left hand side may be treated as a single variable. The right hand side is a definite integral which we propose to differentiate with respect to its upper limit.

Suppose that

$$\int \phi(\varepsilon)\,d\varepsilon = \Phi(\varepsilon)$$

then

$$\int_0^{\varepsilon_c} \phi(\varepsilon)\,d\varepsilon = \Phi(\varepsilon_c) - \Phi(0)$$

Now differentiate with respect to ε_c

$$\frac{d}{d\varepsilon_c}\int_0^{\varepsilon_c} \phi(\varepsilon)\,d\varepsilon = \frac{d}{d\varepsilon_c}\Phi(\varepsilon_c) = \phi(\varepsilon_c) = f_c$$

Therefore

$$\frac{d}{d\varepsilon_c}\left(\frac{T}{bd} \times \frac{\varepsilon_c}{k}\right) = f_c \qquad \text{.....(1)}$$

The differentiation with respect to ε_c may seem a rather arbitrary step. It is necessary because the differential Eq. (1) is a useful equation for our purpose whereas the preceding integral equation is not.

Once T has been determined, this equation can be used to find f_c at every stage in the test. Plotting f_c against ε_c we obtain the stress strain curve for the concrete. However, it is first necessary to find T.

Now take moments about point O (Fig. 1) and equate the total internal moment due to concrete stress and steel force to the external moment M,

$$M = T(d - kd) + b\int_{x=0}^{x=kd} fx\,dx$$

Substituting for x and rearranging

$$\left[\frac{M}{bd^2} - \frac{T}{bd}(1 - k)\right]\left(\frac{\varepsilon_c}{k}\right)^2 = \int_{\varepsilon=0}^{\varepsilon=\varepsilon_c} \varepsilon\phi(\varepsilon)\,d\varepsilon$$

Again we eliminate the integral by differentiating with respect to ε_c

$$\frac{d}{d\varepsilon_c}\left[\left\{\frac{M}{bd^2} - \frac{T}{bd}(1 - k)\right\}\left(\frac{\varepsilon_c}{k}\right)^2\right] = \varepsilon_c\phi(\varepsilon_c) = \varepsilon_c f_c$$

or

$$\frac{d}{d\varepsilon_c}\left[\frac{M}{bd^2}\left(\frac{\varepsilon_c}{k}\right)^2 - \left(\frac{T}{bd} \times \frac{\varepsilon_c}{k}\right)\left(\frac{\varepsilon_c}{k} - \varepsilon_c\right)\right] = \varepsilon_c\frac{d}{d\varepsilon_c}\left(\frac{T}{bd} \times \frac{\varepsilon_c}{k}\right)$$

Expanding the second term on the left hand side and rearranging

$$\frac{d}{d\varepsilon_c}\left(\frac{T}{bd} \times \frac{\varepsilon_c}{k}\right) = \frac{d}{d\varepsilon_c}\left[\frac{M}{bd^2}\left(\frac{\varepsilon_c}{k}\right)^2\right] \Big/ \left(\frac{\varepsilon_c}{k}\right) + \frac{T}{bd}\left(1 - \frac{d}{d\varepsilon_c} \times \frac{\varepsilon_c}{k}\right) \qquad \text{.....(2)}$$

For numerical use Eq. (1) and (2) must be put into finite difference form

$$f_c = \delta\left(\frac{T}{bd} \times \frac{\varepsilon_c}{k}\right) \Big/ \delta(\varepsilon_c) \qquad \text{.....(1a)}$$

$$\delta\left(\frac{T}{bd}\times\frac{\varepsilon_c}{k}\right)=\delta\left[\frac{M}{bd^2}\left(\frac{\varepsilon_c}{k}\right)^3\right]\Big/\left(\frac{\varepsilon_c}{k}\right)+\frac{T}{bd}\left[\delta(\varepsilon_c)-\delta\left(\frac{\varepsilon_c}{k}\right)\right] \quad \text{.....(2a)}$$

Now the top fiber concrete strain ε_c and k have been measured for a series of bending moments M. Thus the first term on the right hand side of Eq. (2a) can be evaluated numerically (for details see the sample calculation). So can the bracketed part of the second term. It has been assumed that the steel force T cannot be obtained by direct measurement and is, in general, unknown. However, a procedure has been described above for determining T at the moment when the concrete cracks start to reopen. At this stage the right hand side of Eq. (2a) can be evaluated completely to give the increase in $(T/bd \times \varepsilon_c/k)$ for a further increase in ε_c of $\delta\varepsilon_c$.

Having the increase δT in the steel force for the strain increment $\delta\varepsilon_c$, the new value of T is used to find the effect of a further strain increment, and so on. In this way T can be traced step by step from the cracking load to the failing load.

At each stage the top fiber concrete stress f_c can be calculated by dividing $\delta(T/bd \times \varepsilon_c/k)$ by $\delta\varepsilon_c$ [Eq. (1a)].

REVIEW OF SOME ASPECTS OF THE METHOD AND ANALYSIS

Before giving a worked example it is convenient to reconsider some points. We have seen that as a starting point in the calculations it is necessary to know the steel force at one point in the load range. In an ordinary reinforced concrete beam the steel force at zero applied moment is only that due to dead weight and shrinkage. For a prestressed beam the steel force can be determined for that bending moment which causes the bending stress in the bottom fiber for the beam to cancel exactly the prestress there. It is assumed that at this load the concrete stress distribution is triangular. This is not necessarily so but for a normal design the assumption is reasonable. However, there is a check. If the distribution is triangular (at this load) then the stress strain curve is a straight line from zero stress and strain up to the stress and strain representing conditions in the top fibers. For larger strains Eq. (1a) is used to determine the stresses. The smooth joining of the stress-strain curve deduced from Eq. (1a) to the straight line assumed for the lower portion provides a check on this assumption (see for example Fig. 5).

After cracking, the steel force is obtained from Eq. (2a), which rests on two assumptions: first, that the concrete stress depends only on the strain, stress history and time effects being ignored; secondly, that plane sections remain plane. The first of these assumptions is debatable and further reference to this is made below. The second follows automatically if symmetrical four-point loading is used. Apart from this the strain distribution is verified experimentally by the strain gage readings. It is important to note that the steel force so obtained is quite independent of any strains which may be measured *in the concrete* adjacent to the steel. If the straight-line distribution above the neutral axis is extrapolated the average concrete strain at the steel level is obtained. This strain can be more or less than the change in steel strain due to load. If there is bond failure causing slip of

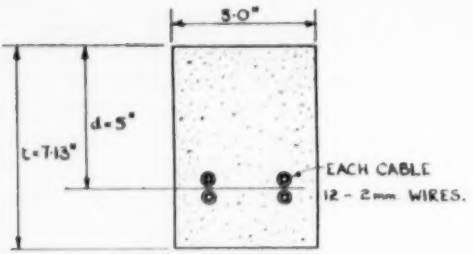

Fig. 2—Cross section of pre-tensioned beam used in example

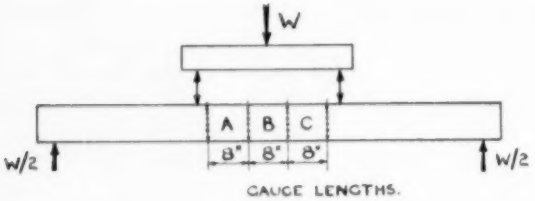

Fig. 3—General layout of loads and strain gages

steel in from the anchorages to the region of high moment the steel strain will be less than the average concrete strain. It is more likely, however, that the effective steel strain will be greater due to local high straining in the immediate vicinity of a crack. Thus Eq. (2a) provides a means of investigating these effects since it deduces the effective steel force.

EXAMPLE

To illustrate the use of this method, results of a test on a pre-tensioned beam will be analyzed. The beam section is shown in Fig. 2. The four cables each consisted of 12 high tensile steel wires of 2-mm diameter. After pre-tensioning the cable ducts were grouted. Fig. 3 gives the general layout of loads and strain gages. A demountable gage was used to obtain the strain distribution over three 8-in. gage lengths: A, B, C. The strain readings are plotted in Fig. 4, which shows

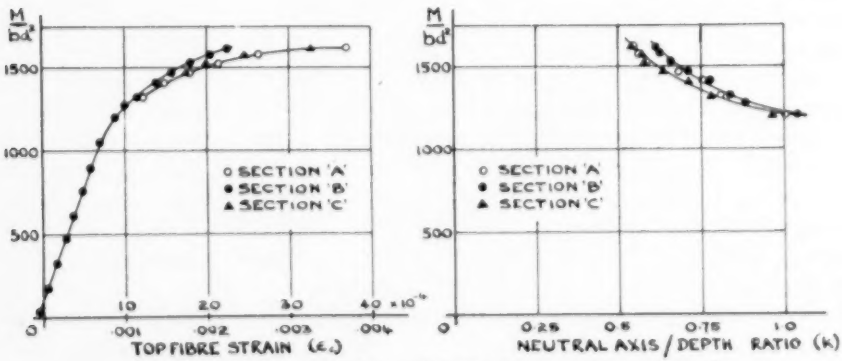

Fig. 4—Top fiber strains and *k* for beam used in example

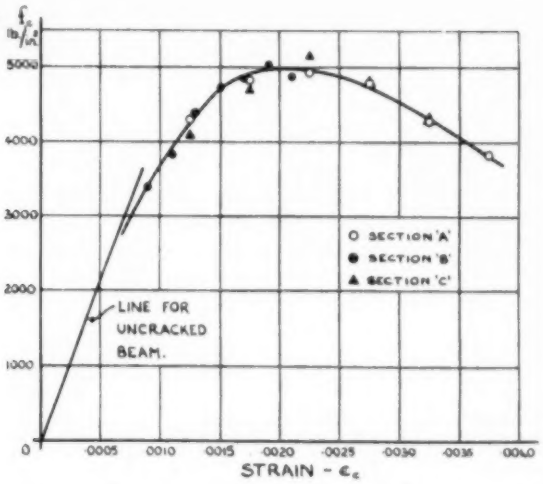

Fig. 5—Stress-strain diagram for beam tested

substantial differences between the readings in the three gage lengths.

The beam failed by concrete crushing, enabling a complete stress-strain curve to be obtained for the concrete. The curve has been calculated from readings in all three gage lengths but detailed analysis will be given only for section A where the greatest strain was recorded. The steps in the calculation are:

(1) Evaluate the prestress strains and add them to the measured strains.

(2) Plot M/bd^2 and k to a base ε_c and draw in smooth curves.

(3) Tabulate M/bd^2 and k for regularly spaced intervals of ε_c (see Table 1).

Table 1 can now be completed down to the line $[\delta\varepsilon_c - \delta(\varepsilon_c/k)]$. The next step is to calculate T/bd for the lowest ε_c by assuming a linear stress distribution; in this case the value 1822 is entered. The corresponding $T/bd \times \varepsilon_c/k$ is entered immediately above. $T/bd[\delta\varepsilon_c - \delta(\varepsilon_c/k)]$ may be calculated for the interval $\varepsilon_c = 10$ to $\varepsilon_c = 15$ by assuming that T/bd remains constant at 1822. The $\delta(T/bd \times \varepsilon_c/k)$ so obtained is added to $T/bd \times \varepsilon_c/k$ for $\varepsilon_c = 10$ to give the value for $\varepsilon_c = 15$. In this way the table is completed column by column. Strictly speaking we should anticipate the increase $\frac{1}{2}\delta(T/bd \times \varepsilon_c/k)$ and use $(T/bd \times \varepsilon_c/k)$ for $\varepsilon_c = 12.5$ in the above calculation; but as δT is comparatively small this refinement is not necessary.

The f_c is calculated last of all and is plotted as in Fig. 5 which also gives values obtained for sections B and C.

CONCLUSIONS

The agreement between the stress-strain curves for the three gage lengths is striking. The same concrete was used so the same curve should be obtained; but at failure, section A showed twice the strain in section B and had a higher neutral axis. This means that section A failed before section B, not because the concrete there was weaker, but because the cracking was more severe due to less effective bond.

TABLE 1

$\varepsilon_c \times 10^4$	10	15	20	25	30	35	40
M/bd^2, psi	1252	1415	1522	1582	1608	1615	1618
k	0.938	0.692	0.611	0.571	0.548	0.545	0.556
$\varepsilon_c/k \times 10^4$	10.7	21.7	32.7	43.7	54.7	64.2	72.0
	16.2	27.2	38.2	49.2	59.5	68.1	
$M/bd^2 \times (\varepsilon_c/k)^2$, psi $\times 10^2$	0.142	0.667	1.63	3.03	4.82	6.66	8.38
$\delta[M/bd^2 \times (\varepsilon_c/k)^2]$, psi $\times 10^2$	0.525	0.963	1.40	1.79	1.84	1.70	
$\delta[M/bd^2 \times (\varepsilon_c/k)^2]/(\varepsilon_c/k)$, psi	3.24	3.54	3.66	3.64	3.09	2.50	
$\delta\varepsilon_c \times 10^4$	5.0	5.0	5.0	5.0	5.0	5.0	
$\delta(\varepsilon_c/k) \times 10^4$	11.0	11.0	11.0	11.0	9.5	7.8	
$\delta\varepsilon_c - \delta(\varepsilon_c/k) \times 10^4$	− 6.0	− 6.0	− 6.0	− 6.0	− 4.5	− 2.8	
$T/bd \times \varepsilon_c/k$, psi	1.94	4.09	6.50	8.97	11.4	13.5	15.4
T/bd, psi	1822	1883	1990	2050	2080	2110	2140
$T/bd[\delta\varepsilon_c - \delta(\varepsilon_c/k)]$, psi	− 1.09	− 1.13	− 1.19	− 1.23	− 0.94	− 0.59	
$\delta(T/bd \times \varepsilon_c/k)$, psi	2.15	2.41	2.47	2.41	2.15	1.91	
f_c, psi	4300	4820	4940	4820	4300	3820	

The same result has been found in other beam tests: crushing has been localized with high strains but analysis of the stress-strain curves has shown no evidence of local weakness in compression.

This point has an interesting consequence. The above analysis assumes that the concrete does not creep. Since this assumption is unjustified the value of the work may be doubted. Consider however the above example. The concrete in section A was strained twice as much as that in section B so it may be assumed that the creep conditions in the two sections were different. Yet, the same stress-strain curve was obtained, indicating that creep does not upset the analysis. This conclusion may not apply if the test is protracted beyond the 4 or 5 hr taken with the above specimen.

ACKNOWLEDGMENT

The test described above is one of a series carried out in 1950 under the direction of Prof. A. L. L. Baker at Imperial College, London. The author is grateful to Professor Baker for permission to use the results.

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2. Prentis, J. M., "The Distribution of Concrete Stress in Reinforced and Prestressed Concrete Beams When Tested to Destruction by a Pure Bending Moment," *Magazine of Concrete Research* (London), No. 5, 1951, pp. 73-77.

3. Prentis, J. M., "A Combined Strain and Curvature Gauge for Use in Testing Concrete Beams," *Journal of Scientific Instruments* (London), V. 32, 1955, pp. 19-21.

For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

CONCRETE BRIEFS

Notes from Field and Office

Inverted Concrete Umbrellas as Ticket Offices 319
Problems and Practices 321

Inverted Concrete Umbrellas Used as Ticket Offices

By MARTIN SCHULZ*

The adaptability of concrete to small structures is well illustrated in four ticket offices built for the National Gymnasium in El Salvador. Designed by architects Ehrentraud Schott and Karl Kastaller, each office consists of an inverted cone supported at the center by a single column (Fig. 1).

The dimensions required were: outside diameter, 33 ft; rise of the cone, 2.9 ft; thickness of the cone, 2⅜ in.; column capital maximum thickness at center column, 8 in. The shape of the column was made as shown in Fig. 2 to comply with the architectural design. Roof drainage was handled by a pipe placed at the center of the column. The outlet was located 4 in. above the footing plate. The footing consists of prolonging the column ribs and plate. Soil pressure calculations had to be made carefully because of earthquake forces and soil conditions.

Design stresses and loads

The analysis was based on shell theory. Analysis of the cone as a membrane gives direct compressive and tensile stresses. These stresses have to be superimposed on stresses due to the edge condition. The latter reduce quickly in the form of damped waves.

Notation of the forces is as follows:

N_ϕ = forces in meridian direction
N_θ = forces in ring direction
M_ϕ = moments due to edge condition

The development of the formulas can be found in references 1, 2, and 3.

For dead load:

$$N_\phi = -\frac{(r_o^2 - r_o^2)g}{r_o \sin 2\phi}$$

$$N_\theta = gr_o \operatorname{cotan} \phi$$

According to the membrane analysis the edge of the cone and the plate would have different deformations. The neces-

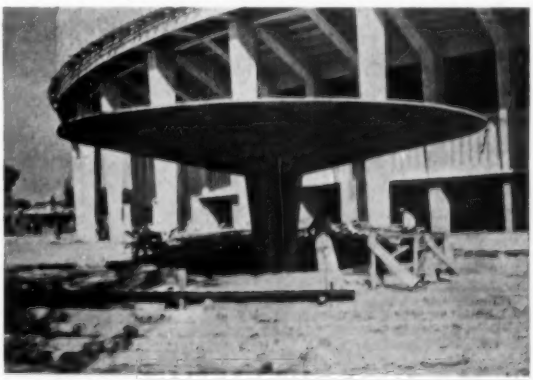

Fig. 1—Stadium ticket office

A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 3, Sept. 1956, *Proceedings* V. 53. Separate prints of the entire Concrete Briefs section are available at 35 cents each. Address 18263 W. McNichols Rd., Detroit 19, Mich.

*Member American Concrete Institute, Structural Engineer, Ministerio de O.O.P.P., Departament de Arquitectura, San Salvador, El Salvador.

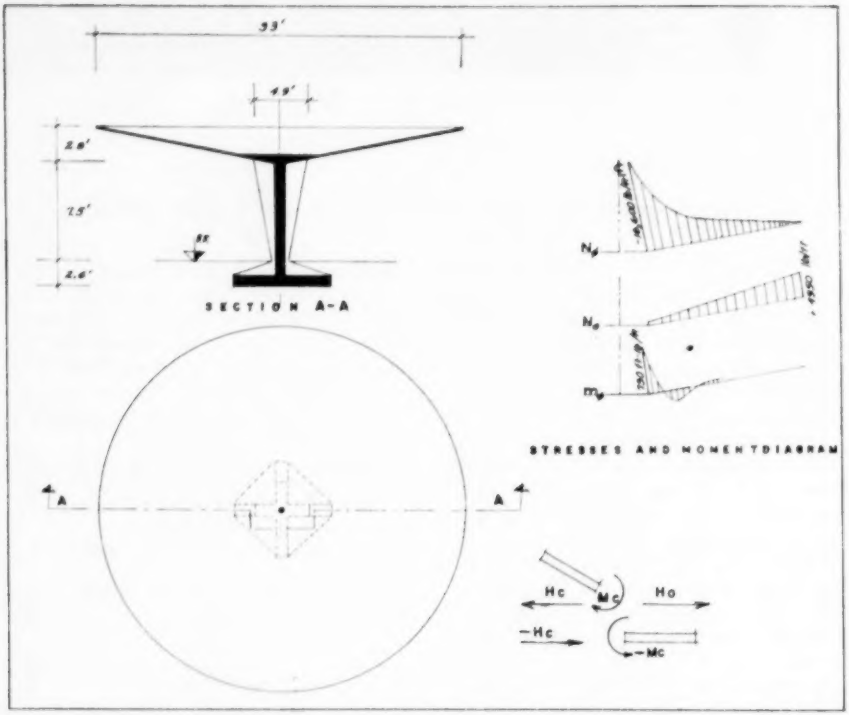

Fig. 2—Plan and elevation, and stresses and moment diagrams

sary alignment conditioned by the monolithic connection creates an edge force, H, and edge moment, M.

Deformations due to the membrane condition are:

$$\delta' = \frac{g}{Es \sin 2\phi}[2r_o^2 \cos^2 \phi + \mu(r_o^2 - r_o^2)]$$

$$\zeta' = \frac{gr_o}{2\,Es \sin^2 \phi}\left[4 \cos^2 \phi - 1 - 2\mu \sin^2 \phi + \left(\frac{r_o}{r_o}\right)^2\right]$$

where δ' is the displacement and ζ' the rotation, E = modulus of elasticity of concrete, s = slab thickness, μ = Poisson's ratio (for concrete 1/6).

Deformations of the cone due to the forces H and $M = 1$ are:

$$\delta_{HC} = \frac{2L^2K \cos^2 \phi}{Es}$$

$$\delta_{MC} = \zeta_{HC} = \frac{6(1 - \mu^2) \sin \phi}{Es^2K^2}$$

$$\zeta_{MC} = \pm \frac{12(1 - \mu^2)}{Es^3K}$$

where $4K^4 = \dfrac{Es \tan^2 \phi}{Dy^2}$

$$D = \frac{Es^3}{12(1 - \mu^2)}$$

Deformations of the plate are:

$$\delta_{HCC} = \frac{(1 - \mu)R}{Es}$$

$$\zeta_{MCC} = \frac{12(1 - \mu^2)R}{Es^3}$$

The edge condition for the shell is:

$$\delta' + \delta_{HC}(H_c + H_o) + \delta_{MC}M_c = -\delta_{HCC}H_c$$

$$\zeta' + \zeta_{HC}(H_c + H_o) + \zeta_{MC}M_c = -\zeta_{MCC}M_c$$

The forces and moments due to the forces H and $M = 1$ are:

$$M_{H=1y} = \sin \phi e^{-Kx} \sin Kx/K$$

$$Q_{H=1y} = e^{-Kx} \sin \phi(\sin Kx - \cos Kx)$$

$$M_{M=1y} = e^{-Kx}(\sin Kx + \cos Kx)$$

$Q_{w-1}y = 2Ke^{-Kx} \sin Kx$

In this case the forces $N\phi$ and $N\theta$ are small and an analysis can be neglected.

If the vertical load is 55 psf, tan ϕ = 0.188, ϕ = 10° 39′, sin 2ϕ = 0.3633, cotan ϕ = 5.317, r_1 = 16.4 ft, cos ϕ = 0.9827, r_0 = 2.46 ft, s = 0.196 ft, E = 3×10^6 psi, y = 2.5 ft, μ = 0.16, the values of H_0 = 11,800 lb per ft and M_0 = 750 ft-lb per ft. With these values the maximum membrane force N_ϕ at the outline of the cone was found to be 5000 lb per ft (tension). The N_ϕ force was found to be a maximum at the column capital with 16,600 lb per ft (compression). Due to the edge condition the maximum moment of 750 ft-lb per ft at the edge of the column capital reduced quickly to 50 ft-lb per ft approximately 3 ft from the column capital.

The column capital is designed for the compressive force N_ϕ and the bending moment M_ϕ.

Ultimate strength of concrete was 3200 psi. The steel had a working stress of 15,600 psi. The concrete was vibrated during placing, especially at the column capital where the steel crisscrossed closely. After the concrete was placed the surface was kept wet constantly, which is an important factor in this climate. No cracks were noticed.

References

1. Santarella, L., *Il Cemento Armato*, Ulrico Hoepli, Milan.

2. Girkmann, K., *Flaechentragwerke*, Springer-Verlag, Vienna, 1954, 558 pp.

3. Beyer, K., *Die Statik im Stahlbetonbau*, Springer-Verlag, Berlin, 1955, 804 pp.

Problems and Practices

A series relating to "down-to-earth, everyday" concrete problems which attempts to give brief answers to the more common (and sometimes uncommon, too) questions asked about concrete design and construction practices.

To some, the answers will seem simple and obvious; others may prove to be extremely controversial.

All ACI members are invited to participate—either by submitting an inquiry, or even better, by telling JOURNAL readers how an intriguing problem was solved. It may well be that readers will be able to suggest more practical solutions than those presented.

Q. How can one avoid shrinkage cracking in plastic concrete, before, during, and immediately after finishing? The condition occurs in summer days when temperatures are high, humidity low, and particularly in combination with drying wind.

A. A fine fog spray of water applied immediately on concrete flat slabs after finishing has proved successful in eliminating shrinkage cracks. This applies particularly to slabs on the ground, where no sand cushion or similar absorbent material has been provided underneath; tilt-up slabs cast on concrete slab; or upper floors where concrete is placed directly on steel pans or nonabsorbent forms of plywood or similar material.

Some years ago when curing compound or membrane curing was first developed to take the place of water curing, comparative tests were made in the Imperial Valley, Calif., during the hot summer months, and small loss in compressive strength was noticeable compared with conventional water curing. Accordingly, because of its convenience and lesser cost, it was gradually adopted in practically all specifications.

Under certain weather conditions, it has been discovered that shrinkage cracks in concrete slabs began to appear directly after, and sometimes before, the finishers were through with

their final finish. For floor slabs placed on the ground it was found that a sand cushion of 2 in. underneath the concrete eliminated most, if not all, cracks. However, with tilt-up slabs being cast directly on the concrete floor and in the upper concrete floors of multistoried buildings, shrinkage cracks appeared with annoying frequency in spite of the brand of curing compound used.

After much experimentation with wet and dry concrete, early and late finishing, and other ideas, the cracks continued to appear. The only thing left was to go back to water curing immediately following the final finish.

A fine fog spray, such as is used by the irrigation concrete pipe industry, proved to cure the condition. The curing compound is applied in the usual manner the second day, and satisfactory results are obtained.

The fine spray applied to the concrete is also helpful to the finishers when weather conditions cause it to set up fast and become gummy or sticky, making it difficult to finish.

Portland cement stucco suffers by rapid evaporation of water at the early stages, particularly the walls exposed directly to the sun. A fine spray of water applied to the several coats as soon as possible after the plasterer has finished will likewise prevent most, if not all, shrinkage cracks. With such a fog spray, there is sufficient water to replace the water which disappears by rapid evaporation, but not enough to wash out the cement from the mix.

J. E. JELLICK, Manager and Civil Engineer, Portland Cement Information Bureau, San Francisco

* * * *

Q. *What is the purpose of the common specification requiring a 2-hr delay after casting a concrete column before placing the slab it supports? Is this 2-hr wait the minimum time?*

A. Little research has been conducted on this problem, so there is no formula relating vertical settlement of concrete to (1) temperature, (2) slump, (3) setting time of cement, and (4) form material. ACI Committee 611 in the *ACI Manual of Concrete Inspection* recommends a 2-hr delay. The ACI Building Code does not contain any requirement for such a delay, and the delay required by various engineers' specifications varies from none to 24 hr so that contractors tend to regard the delay as a whim of the specification writer.

A report by ACI Committee 605, Settlement and Shrinkage of Concrete in Forms, in 1931 presented some interesting test data. The results justify a 2- to 4-hr wait for settlement using ordinary cement in the concrete at usual placing temperatures. An abstract of this report is reproduced here.

Although there is available much data on the results of laboratory tests on shrinkage of concrete at the age of 24 hr or more, and the behavior of concrete while in the plastic state in the molds has been studied in the laboratory, there is little information from actual field tests for settlement and shrinkage during the 24 hr after placing.

The purpose of these tests was to determine the settlement and shrinkage of concrete after being placed in the forms. The tests were conducted under actual field conditions on a large reinforced concrete warehouse of the flat slab type during its construction at Long Island City, N.Y. Additional tests were made at a later date on a reinforced concrete department store building in the course of construction at Jamaica, Long Island, N.Y. All tests were made on 11 ft long columns.

Because the lapse of time between the casting of the concrete columns and the supported slab above may not only affect the quality of the structure, but often is a factor influencing the cost and speed of the whole operation, it was hoped that the results might be a step in determining a minimum standard time interval between the casting of the columns and their supported slab.

In each group of tests, to determine

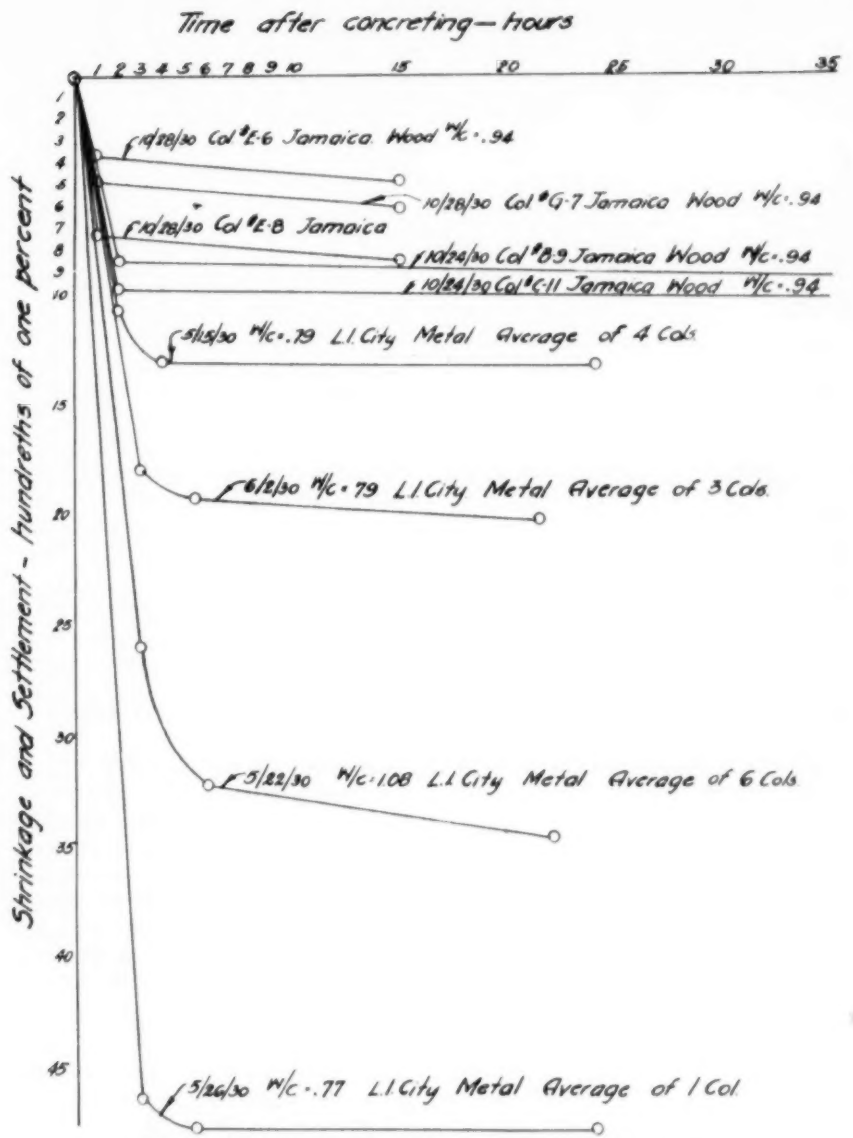
Time after concreting—hours
1 2 3 4 5 6 7 8 9 10 15 20 25 30 35
Shrinkage and Settlement - hundredths of one percent
10/28/30 Col #E-6 Jamaica Wood W/c .94
10/28/30 Col #G-7 Jamaica Wood W/c .94
10/28/30 Col #E-8 Jamaica
10/24/30 Col #B-9 Jamaica Wood W/c .94
10/24/30 Col #C-11 Jamaica Wood W/c .94
5/15/30 W/c .79 L.I. City Metal Average of 4 Cols.
6/12/30 W/c .79 L.I. City Metal Average of 3 Cols.
5/22/30 W/c .108 L.I. City Metal Average of 6 Cols.
5/26/30 W/c .77 L.I. City Metal Average of 1 Col.

Fig. 1—Shrinkage and settlement in concrete columns

the settlement and shrinkage of the concrete after final rodding in place in the column forms, a number of columns were placed to the underside of the floor and after the workmen had leveled off the surface, measurements were taken.

Conclusions

The results of both groups of tests show a settlement and shrinkage up to 15 hr varying from 0.05 to 0.47 percent. However, an inspection of the curves of Fig. 1, shows that the contraction be-

TABLE 1—CEMENT TESTS

Job	Brand of portland cement	Fineness percent passing No. 200 sieve	Setting time, hr		Tensile strength, 1 cement : 3 sand, psi	
			Initial set	Final set	7 days	28 days
Long Island City	Y	85.8	3.10	6.00	313	401
Long Island City	Y	84.7	3.22	5.37	312	397
Long Island City	Y	84.4	3.12	6.27	298	395
Jamaica	X	87.2	4.08	6.13	347	440
Jamaica	X	87.4	4.08	6.13	357	425
Jamaica	X	87.4	4.03	6.06	347	425

tween 4 and 15 hr, with the exception of one test, never exceeded 0.012 percent or about 1/64 in.

The variations during the first 4 hr were not entirely unexpected and were due to difference in manipulation, rodding, absorption, evaporation, temperature, water-cement ratio, and slight leaks in forms. Also, a column which was partly filled, lacking perhaps one buggy, which stood 15 minutes before filling was completed, would unquestionably show less early settlement than a column which lacked, for instance, three buggies and stood only 10 min before being completely filled. Further, due to working conditions the time from the filling of the columns to the initial readings varied slightly, as the initial reading could not be taken until the workmen who were cleaning the steel and forms of surplus concrete had finished their job.

The curves also show that the pronounced contraction continued longer on the Long Island City job than on the Jamaica operation. The contraction in the wood forms up to 2 hr is much less than in the metal forms and becomes small, from a field standpoint, after that time. The initial changes continue in the metal forms for about 4 hr.

Allowing for the tests being made on different jobs, it would seem to one familiar with both operations that there are not sufficient reasons to ascribe the wide variation between the two groups to methods of procedure, as practically the same concrete gang was employed during the construction of both buildings, using similar systems of handling the concrete. An examination of the mill tests of the cement (Table 1) do not offer a solution as the setting time is consistently earlier for the first group. Furthermore, the air temperatures during placing would indicate an earlier settlement for the first groups, other conditions being the same.

In his laboratory tests of portland cement mortars, J. C. Pearson* found in his experiments with absorptive forms that the initial volume changes which had been so marked when using nonabsorbent bases were greatly reduced in practically all cases.

There is a certain similarity between these results and those obtained using the metal forms (Long Island City job) and the wood forms (Jamaica job). The metal forms probably allow for little loss of water other than by evaporation, whereas the absorption by the wood forms would reduce the settlement and shinkage of the concrete. However, further tests will have to be made to draw definite conclusions.

It is felt that the most interesting result of the experiment is shown in Fig. 1 indicating that after the fourth hour the shrinkage in columns is small and nearly enough the same in all columns to be considered uniform from a field standpoint.

*Pearson, J. C., "Shrinkage of Portland Cement Mortars and Its Importance in Stucco Construction," *Proceedings*, ACI, V. 17, 1921, pp. 133-148.

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Bridges

Repairing the James River bridge system

GEORGE VACCARO, *Consulting Engineer*, V. 7, No. 4, Apr. 1956, pp. 46-50
Reviewed by ARON L. MIRSKY

Description of damage to concrete trestle piles and caps on Virginia's Route 17, the repairs, and the cost-plus contract arrangement. Damage to concrete is ascribed to action of the salt in the sea water, freezing and thawing, and abrasive action of the water. Cost-plus contract was awarded on basis of some competitive factors.

Highway bridges in the United States (Strassenbruecken in USA)

W. KLINGENBERG, *Der Bauingenieur* (Berlin), V. 30, No. 4, Apr. 1955, pp. 117-122
Reviewed by ARON L. MIRSKY

A German bridge engineer gives his impressions: American practice is technically rather backward, but its solution of the problem of traffic is well handled, and the bridges are increasingly becoming aesthetically satisfying.

The two "Bytown" prestressed bridges at Ottawa (Les deux ponts precontraints "Bytown" a Ottawa)

HANS-PETER KAEGI, *L'Ingenieur* (Montreal), V. 41, No. 164, Winter 1955, pp. 11-14
Reviewed by ARON L. MIRSKY

Details of design and construction of two bridges, each of three spans, over the Rideau River at Bytown, Ottawa (see also *The Engineer*, Mar. 18, 1955, p. 391; "Current Reviews," ACI JOURNAL, Nov. 1955, *Proc.* V. 52, p. 385.

Construction

Composite construction makes sense

GILBERT D. FISH, *Consulting Engineer*, V. 7, No. 5, May 1956, pp. 51-55
Reviewed by ARON L. MIRSKY

Use of composite construction for buildings can lead to substantial savings in steel, especially if cover plates are used and the steel members are shored until the concrete has attained desired strength and composite section takes dead as well as live load. Data are given for IBM Engineering Laboratory at Poughkeepsie, N. Y., constructed in 1955, using welded steel studs as shear connectors.

Uses of aerated cement grout and mortar in stabilization of slips in embankments, large-scale tunnel repairs, and other works

MAXWELL C. PURBRICK and DOUGLAS J. AYRES, *Proceedings*, Institution of Civil Engineers (London), Part II, V. 5, No. 1, Feb. 1956, pp. 52-84 (including discussion)
Reviewed by ARON L. MIRSKY

After considering production and properties of aerated "concrete," authors describe its use in stabilizing track ballast and slopes, repairing tunnels, and grouting viaduct piers on the Western Region of British Railways.

Construction of a new cement plant at Edmonton

W. RUTHERFORD, *The Engineering Journal* (Montreal), V. 39, No. 4, Apr. 1956, pp. 412-420
Reviewed by ARON L. MIRSKY

Description of new (1955) wet-process plant in Alberta, Canada. Plant layout is featured; design and other considerations are well covered. Construction details are discussed briefly.

Dams

Symposium on arch dams

Proceedings, ASCE, V. 82, PO 3—*Journal of the Power Division*, June 1956, 224 pp.

Includes nine papers on arch dams presented at the ASCE Symposium on Arch Dams, June, 1956, at Knoxville, Tenn. The various papers present design and construction problems on no-

table arch dams of different types in several countries. Theory of arch dam design is included in several of the papers.

Raising and strengthening of the Steenbras Dam

SOLOMON S. MORRIS and WILLIAM S. GARRETT, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 1, Jan. 1956, pp. 23-55 (including discussion)

Reviewed by ARON L. MIRSKY

Rapid augmentation of Cape Town, South Africa, water supply was obtained by raising Steenbras Dam 6 ft. Analysis showed existing mass concrete spillway section to be unstable under 5.5 ft surcharge, but stable under 3 ft (latter being confirmed by past experience under flood flows). Coyne method of anchoring existing dam to rock foundation via post-stressing was found to offer many advantages in terms of cost, speed, and possibility of later additional increases in height and was adopted.

See also article by senior author in *Civil Engineering*, V. 26, No. 2, Feb. 1956, pp. 33-37.

Design

Stress analysis in frames (in Spanish)

E. TORROJA, Publication No. 159, Instituto Técnico de la Construccion y del Cemento (Madrid), 1954, 60 pp.

Reviewed by E. ROSENBLUETH

This is the fifth instalment of a series on structural analysis by the same author. Previous publications of the series dealt with fundamentals of plane structures with straight members (No. 71), stress analysis in straight bars (No. 100), elementary truss analysis (No. 101), and stress analysis in curved members (No. 103). In the present publication Torroja describes the method of moment distribution and an approximation thereto through use of partial end-restraint coefficients. He generalizes the latter to beams on elastic supports, sidesway, and Vierendeel trusses. A number of graphs and monograms facilitates application to members of variable cross section. The method proposed is useful in that it gives a good rapid approximate solution for most cases encountered in practice. However, the text is difficult to follow despite the profusion of excellent figures, for, in an effort toward extreme conciseness, the author employs a number of unusual notations and neologisms. Similarly, much is lost through definition of notations without reference to their physical significance. There is no index or bibliography. The profession would profit from a more readable version of the approximate method proposed by the author.

Design of lightweight reinforced concrete (Dimensionierung bewehrter Leichtbetons)

O. CAMEHN, *Betonstein-Zeitung* (Wiesbaden), V. 21, No. 10, Oct. 1955

Reviewed by WERNER H. GUMPERTZ

Discusses structural design procedures and formulas for aerated concrete, volcanic ash concrete, and other lightweight concretes. Special attention is given to the effect of variations in the modular ratio n. Graphs facilitating design are presented.

The analysis of structures

N. J. HOFF, John Wiley and Sons, Inc., New York, 1956, 493 pp., $9.50

An attempt "to make the study of structures an exciting undertaking by developing a unifying standpoint to show that methods used by structural engineers are all logically interconnected." Begins with the presentation of structural analysis as a logical and unified theory based on a small number of first assumptions. Develops the principle of virtual work, minimal principles of structural theory, and buckling. Examples and applications suitable for advanced problems in aeronautical, civil, and mechanical engineering, and applied mechanics are included. Wherever mathematical methods beyond elementary calculus are used, they are explained to allow the reader to proceed without reference to other books. Footnotes have been generally collected and included in the appendix which also

serves as an excellent bibliography. The book is well written and up-to-date in the applications and examples presented. Suitable for a text on graduate study of structural engineering or for a practicing engineer's reference library.

A general method for the analysis of grid frameworks

ARNOLD W. HENDRY and LESLIE G. JAEGER, *Proceedings*, Institution of Civil Engineers (London), Part III, V. 4, No. 3, Dec. 1955, pp. 939-971

Reviewed by ARON L. MIRSKY

Presents a method for the analysis of open grid frames with members rigidly connected and supported on two, three, or four sides. Torsional effects are included. Method involves solution of a system of differential equations by harmonic analysis. Theoretical and experimental results are compared.

Approximate analysis of concrete domes (in Spanish)

GUNHARD ORAVAS, *Revista del Colegio de Ingenieros de Venezuela* (Caracas), No. 240, Mar. 1956, pp. 34-42

Reviewed by JOSEPH J. WADDELL

Exact mathematical analysis is difficult or impossible for certain thin-shell structures. In the case of certain self-supporting symmetrical shells or shells of rotation, an approximate method is proposed which gives satisfactory results. The method consists of replacing the shell structure with a conoid structure which is tangent to the shell and of the same thickness. The application of differential geometry permits determination of the required properties.

What wind intensity should be considered for design of structures? (in Serbian)

MARKO RADOJKOVIC, *Naše Gradevinarstvo* (Belgrade), No. 1, 1956, p. 33

Reviewed by J. J. POLIVKA

Criticism of Par. 23221 of the Yugoslav building code, PTP 2, which under certain circumstances would reduce the wind load 17.5 psf (90 kg per sq m) for buildings from 33 to 99 ft (10 to 30 m) high to a small intensity of 5 psf (25 kg per sq m). Reference is made to German standard regulations DIN 1055 specifying wind load of 16.5 psf (80 kg per sq m) for buildings up to 63 ft high and 22.5 psf for buildings up to 328 ft (100 m) high, and to American AISC specifications of approximately the same intensity.

An extended load-splitting method for simplified analysis of plates (Ein erweitertes Lastaufteilungsverfahren zur vereinfachten Plattenberechnung)

HUBERT BECK, *Der Bauingenieur* (Berlin), V. 30, No. 4, Apr. 1955, pp. 150-154

Reviewed by ARON L. MIRSKY

An approximate method is developed for the analysis of rectangular plates variously supported on three edges and free on the fourth, under various types of distributed loads. As numerical examples, following cases are investigated: (a) simply supported on three edges, uniformly loaded, with length/breadth ratios of 2.0, 1.5, 1.0, and 0.5; and (b) simply supported on two opposite edges, encastre on the third, line loading on the free edge—*e.g.*, an intermediate landing in a flight of stairs. Results are close to those obtained by exact (theory of elasticity) methods.

A more exact method for calculating the flexural vibrations of portal-type structures (Ein genaueres Verfahren zum Berechnen der Biegeschwingungen portalfoermiger Bauten)

J. GEIGER, *VDI Zeitschrift* (Duesseldorf), V. 98, No. 7, Mar. 1, 1956, pp. 261-266

Reviewed by ARON L. MIRSKY

Method presented in author's 1922 paper in same journal, which considered only vibrations of fundamental frequency, is extended to include the first harmonic, to increase accuracy required for design of such structures as turbine foundations under present-day higher rotational velocities and loadings. Good correlation is shown to exist between calculated and experimental values.

Design of round silo cells for cement storage (Zur Berechnung runder Silozellen fuer Zementlagerung)

FRANZ BOEHM, *Beton- und Stahlbetonbau* (Berlin), V. 51, No. 2, Feb. 1956, pp. 29-36

Reviewed by VALDIS LAPSINS

Design of silos in general and principles of wall design of the cylinders is given first consideration. Then actual design of walls is discussed for storage of cement in powder form and air-mixed cement. Several numerical examples illustrate the design principles. Continuation of the subject is found in *Beton- und Stahlbetonbau*, V. 51, No. 3, Mar. 1956, pp. 59-62. This part handles the vertical reinforcing of the cylinders as well as the design of the cell base.

Foundations—Design and practice

ELWYN E. SEELYE, John Wiley and Sons, Inc., New York, 1956, 450 pp., $16

An addition to the series of data books for civil engineers by Mr. Seelye. Provides a comprehensive manual for the design of foundations. Includes a minimum development of theory to accompany the design and practice recommendations. The recommendations on practice are presented in a particularly effective way through hundreds of carefully prepared drawings showing common defects and failures in buildings and the proper methods of design and detailing to avoid these defects. The book should be of great assistance to all practicing structural engineers because of its variety of practical solutions for foundation problems in design and analysis, specifications, and construction and inspection procedures.

Materials

Use of blast-furnace slag as a concrete aggregate

ERIC F. FARRINGTON, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 1, Jan. 1956, pp. 56-59

Reviewed by ARON L. MIRSKY

Examination of structures made with blast furnace slag concrete and dating from around 1910 has shown the concrete to be sound and noncorrosive to embedded steel. Recent construction of foundations at the Appleby-Frodingham Steel Co., using slag produced in its own mills, is described. A brief summary of the characteristics of the slag is included.

Grading and proportioning for quality concrete block

WILLIAM GRANT, *Concrete*, V. 64, No. 2, Feb. 1956, pp. 30, 34, 36, 40

Discusses general requirements, qualities, and treatment of aggregates to produce concrete masonry most economically—the objective, of course, being to produce a block of satisfactory strength and other qualities with a minimum amount of cement. For a more complete treatment of manufacturing problems the author refers readers to his book *Manufacture of Concrete Masonry Units*.

Applications of high-alumina cement

P. H. HARRIS, *The Engineering Journal* (Montreal), V. 39, No. 3, Mar. 1956, pp. 219-222

Reviewed by ARON L. MIRSKY

Properties and uses of aluminous cements, with especial emphasis on their characteristics as castable refractories.

The interested reader may also be referred to "Les betons refractaires," by L. Longchambon, *Memoires*, Societe des Ingenieurs Civils de France, V. 108, No. 2, Mar.-Apr. 1955, pp. 135-156; "Current Reviews," ACI JOURNAL, Dec. 1955, *Proc.* V. 52, p. 498.

Some remarks on the use of crushed blast furnace slag (Quelques reflexions au sujet de l'emploi des laitiers concassés de haut fourneau)

H. MANCHE, *Revue des Matériaux de Construction* (Paris), No. 481, Oct. 1955, pp. 252-255

Reviewed by PHILLIP L. MELVILLE

From a series of tests on concrete with slag and igneous aggregates, similar strength results were obtained. In view of the variations encountered in crushed slag it is necessary to use larger size test samples to offset local variations. For optimum results it is advisable to use fairly heavy slags, and to saturate them just before use.

Unsatisfactory ferric cements and Ferrari cements (Ciments ferriques défectueux et ciments Ferrari)

F. Ferrari, *Revue des Matériaux de Construction* (Paris), No. 484, Jan. 1956, pp. 17-22
Reviewed by Phillip L. Melville

Reviews the proper process for the successful manufacture of Ferrari cements. Some high-iron cements have given bad results due to improper fabrication; lack of homogeneity results in nonuniform burning and improper Al/Fe ratio. Thirty-two references are addended.

1955 Book of ASTM Standards. Part 3—Cement, concrete, ceramics, thermal insulation, road materials, waterproofing, soils

American Society for Testing Materials, Philadelphia, 1956, 2062 pp., $13.50

The triennial publication of the ASTM Book of Standards. Concrete technicians and users will particularly note new tentative standards for portland pozzolan cement, false set of portland cement, and flexural strength of hydraulic cement mortars. Tentative standards are also included for the use of fly ash as an admixture in portland cement concrete and the ball penetration test for consistency of fresh portland cement concrete.

Pavements

Use of special cements for concrete road pavements (Zur Frage von Sonderzementen fuer den Strassendeckenbau)

W. Humm, *Zement und Beton* (Vienna), V. 1, No. 4, Feb. 1956, pp. 3-7
Reviewed by Fritz Kramrisch

Presents a general discussion and comparison of different test programs in the United States and Germany regarding the influence of the type of cement on the behavior of concrete road pavements. Further improvements of the quality of cement chemically and mineralogically seem to be limited by the possibilities of variations between the three main elements CaO, SiO_2, and Al_2O_3. Of interest is a remark regarding a test program in progress to counteract the cracking, not by reducing the shrinkage, but through increase of the ductility of the cement. It seems that this characteristic can be developed and improved through a higher homogeneous and mineralogically well defined product, with a harmonically controlled grain size.

Progress in sawed contraction joints

F. Kypreos, *Contractors and Engineers*, V. 53, No. 6, June 1956, pp. 36-41

Describes methods of sawing contraction joints in concrete pavement with special consideration of cost factors. Reports that this procedure is now required by 19 states, optional in six, and under experimentation in six. Concludes with a short description and discussion of the joint sawing equipment available from some 17 manufacturers.

Precast concrete

Precast concrete

Architectural Record, V. 119, No. 6, June 1956, pp. 215-219

A survey of current U. S. practice with emphasis on prestressing. Thirty companies, fabricating precast (including prestressed and ordinary reinforced) concrete, were included in the survey. A brief description of the products available and processes used is included. The illustrations of different fabrication methods and erection of different precast systems are particularly good. The survey includes notes on types and sizes of units available, areas served, and the size of the market.

An original method of draining unwalled traffic tunnels using reinforced concrete drain elements (in Serbian)

Dragomir Dimitrijevic *Naše Gradevinarstvo* (Belgrade), No. 1, 1956, pp. 38-40
Reviewed by J. J. Polivka

Description of "La Graou" road tunnel in the French Alps, built in solid rock. To eliminate dripping water, precast concrete arch shells were installed. Spanning 24 ft in units 3 ft 3 in. wide, the shells rest on continuous concrete

beams 11 ft 6 in. above the road with drainage holes inside. The shell units are 2 in. thick and stiffened on each side with arch ribs of 2 3/8 to 3 in. section. Concrete drainage ducts 1 ft 9 in. wide are also provided under sidewalks.

Precast concrete building frame

The Engineer (London), V. 201, No. 5221, Feb. 17, 1956, pp. 219-220

Reviewed by ARON L. MIRSKY

A two-story addition was constructed over an existing three-story furniture warehouse in Acton, England, primarily of precast elements. The columns were mainly two stories in length, with an independent welded lattice cast in and left exposed at the level of the intermediate floor; the beams were then joined to this lattice so no haunches were required. Main beams were formed of a combination of precast prestressed members and cast-in-place concrete, the former being the tension and the latter the compression zone.

Prestressed concrete

Design of prestressed concrete beams for ultimate strength

H. J. COWAN, *Constructional Review* (Sydney), V. 29, No. 2, Feb. 1956, pp. 18-29

Classifies failure of prestressed concrete beams according to three modes: (1) primary tension failure with steel rupture; (2) primary tension failure without steel rupture; and (3) primary compression failure. Develops equations to express the ultimate capacity of beams, both rectangular and I-shaped, for the three modes of failure.

Wire and strand for prestressed concrete

H. K. PRESTON, *Civil Engineering*, V. 26, No. 6, June 1956, pp. 46-49

Discusses commercially available steel (in the United States) for both pre-tensioned and post-tensioned prestressed concrete. Includes recommendations for properties required in each type of work and a good description of the properties of the various commercially available materials.

New steels for prestressed concrete construction (Neue Spannstaehle fuer Spannbetonkonstruktionen)

P. WINCIERZ, *VDI Zeitschrift* (Duesseldorf), V. 98, No. 9, Mar. 21, 1956, p. 385

Reviewed by ARON L. MIRSKY

Abstract of article by F. Lickmeier in *Draht*, V. 6, 1955, pp. 410-414, describing properties of Sigma steels developed in West Germany.

Can we design prestressed concrete by allowable stresses?

T. Y. LIN, *Journal of Prestressed Concrete Institute*, V. 1, No. 1, May 1956, pp 3-5

Discusses criteria for design and pitfalls when designing by the elastic theory. Concludes that elastic design on the basis of a fixed set of allowable stresses is entirely unsatisfactory for prestressed concrete. Suggests that a flexible set of values be required to guide engineers but not restrict them, and that structures so designed be checked for factor of safety at ultimate strength. Engineers should design prestressed concrete on the basis of its behavior and strength and not on the basis of arbitrary stress requirements.

Curing methods and duration studies of pre-tensioned units

A. M. OZELL and W. D. GIVENS, *Engineering Progress at the University of Florida*, V. 10, No. 3, Mar. 1956, 8 pp.

Compressive strength, modulus of elasticity, and bonding properties of the concrete on pre-tensioned reinforcement were measured for both moist curing and steam curing up to 7 days age. The conclusions obtained were that sudden release of strand should be avoided; that minimum wet-curing time of 5 days at 70F and of steam curing time of 3 days at 130F is required to produce approximately 75 percent of the design strength of the Type I cement used. It was also concluded that approximately a 3000 psi cylinder strength and 3×10^6 psi modulus of elasticity are required for the concrete to reach acceptable bond and compressive strengths to enable stress releasing on the pre-tensioned reinforcement.

Properties of concrete

Capillarity of manufactured concrete (La capillarité des bétons manufacturés)

L. Marill. *Revue des Matériaux de Construction* (Paris), No. 485, Feb. 1946, pp. 31-40.
Reviewed by Phillip L. Melville

The sundry causes of water absorption are reviewed: influence of mixing water, amount of cement, variations in aggregate gradation, compaction and vibration, curing. A series of waterproofing agents were also studied (wax, bitumen, fluosilicates, latex). It is concluded that to lower capillarity a minimum amount of mixing water is needed, also good gradation and curing. Certain waterproofing agents may also help. The article concludes with a theoretical study of capillarity.

Defining concrete mixes

Engineering (London), V. 181, No. 4697, Feb. 3, 1956, p. 125

International definition of concrete mixes

The Engineer (London), V. 201, No. 5217, Jan. 20, 1956, p. 105
Reviewed by Aron L. Mirsky

Reprinted from "Recommendations of the International Subcommittee on Concrete for Large Dams on Definition of Concrete Mixes Used in Dam Construction" Paper No. 11 of the Report, Fifth Congress on Large Dams, Paris, 1955. To permit published descriptions of concrete to be compared intelligently, report recommends items to be included in (a) minimum reports, and (b) full reports.

Wood, steel and concrete in fires (Holz, Stahl und Beton im Brandfalle)

H. Friedrichs, *Die Bauzeitung* (Stuttgart), V. 61, No. 3, Mar. 1956, pp. 94, 96
Reviewed by Aron L. Mirsky

Heat resistance rather than flammability is recommended as criterion of fire safety in selection of materials for load-carrying structural elements (walls, floors, stairs, roofs).

Ocrating concrete for greater strength and greater resistance to chemicals (in Dutch)

B. P. Hofstede, *Polytechnisch Tijdschrift* (The Hague), V. II, No. 7-8, Feb. 16, 1956, pp. 136b-139b
Reviewed by John B. Snethlage

Describes a process whereby a great variety of building materials, such as reinforced concrete poles, sewage pipe, tiles, and bricks, can be made stronger and more resistant to weather and chemical corrosion. The method requires exposing the finished product, at the time of final set of the concrete, to (SiF_4) in a pressure vessel or reactor, which is said to cause deep penetration of the gas and result in a hard, dense, and resistant outer layer and surface. Tests are described with photos of comparative results. Basic principles and chemical reactions are explaind.

Accelerated curing of concrete test cubes

T. N. W. Akroyd and R. G. Smith-Gander, *Engineering* (London), V. 181, No. 4699, Feb. 17, 1956, pp. 153-155
Reviewed by Aron L. Mirsky

Freshly cast cubes are placed, molds and all, in hot water (about 50 C) which is then brought to a boil and kept boiling for a minimum of 7 hr. Cubes are then taken out, unmolded, and tested in the usual manner, within 1 hr after being taken from the boiling water. Elapsed time between casting and immersing in the hot water, initial water temperature, and maximum boiling time were found to be relatively noncritical, but results were quite sensitive to elapsed time between end of boiling and testing. Relation between results of accelerated tests and 7- and 28-day tests are given in graphical form.

Concrete an aeolotropic material

L. B. Mercer, *Commonwealth Engineer* (Melbourne), V. 43, No. 6, Jan. 2, 1956, pp. 183-188

Presents the results of tests which show that concrete is nonisotropic and nonhomogeneous. Qualities investigated were compressive strength as affected by aggregate orientation; compressive

strength and permeability as affected by direction of compression or percolation in relation to direction of casting. The author concludes that concrete is an aeolotropic material with a 10-20 percent greater compressive capacity in the direction of casting than perpendicular thereto. He also concludes that as compression test specimens, cubes tested on their side give a more realistic appraisal of the compressive capacity of concrete for compression in horizontal flexural members than do standard cylindrical specimens. He further concludes that the conversion factors for equating the results of cube tests and cylinder tests combine the factor of aeolotropy with the effect of the shape of the specimen.

Structural research

Plastic distortions on the Nibelungen bridge at Worms (Plastische Verformungen an der Nibelungenbruecke Worms)

A. MEHMEL and T. FEY, *Der Bauingenieur* (Berlin), V. 30, No. 3, Mar. 1955, pp. 90-93

Reviewed by ARON L. MIRSKY

Three types of long-term readings were taken: (1) short-gage-length strain readings in the concrete webs over the piers; (2) deck levels; and (3) long (30m) gage length strain readings on two cantilever arms.

For a description of the bridge, see H. Seidel, *Zeitschrift VDI*, V. 95, No. 16, June 1, 1953, pp. 480-481; "Current Reviews," ACI JOURNAL, Jan. 1954, *Proc.* V. 50, p. 397.

No shear keys are needed here

W. E. DEAN and A. M. OZELL, *Engineering News-Record*, V. 156, No. 23, June 7, 1956, pp. 61-64

Describes a test to failure of a precast, prestressed concrete beam with a cast-in-place slab as top flange. A full-size section suitable for highway bridge use was employed. Purpose of the research was to answer the question: are shear keys necessary to resist horizontal shear between beam and slab in composite concrete beam and slab construction? Shear keys were omitted and no mechanical anchorage other than vertical loops sufficient for the vertical component of diagonal tension were employed. Ultimate load was 12 percent higher than that calculated by the formula in "Criteria for Prestressed Concrete Bridges" of the Bureau of Public Roads. The maximum computed horizontal shear resistance was 265 psi. No movement whatsoever was detectable. The authors suggest a code provision allowing a horizontal shearing stress across the joint in a composite section of 0.06 f_c' with a maximum of 210 psi to correspond to the AASHO specification for bond on top bars to apply for the case of composite construction without shear keys but with tie-down steel.

Dynamic behavior of elastic structures with fixed ends (Das dynamische Verhalten von elastisch eingespannten Bauwerken)

G. LOHSE, *VDI Berichte* (Duesseldorf), V. 4, 1955, pp. 57-60

Reviewed by ARON L. MIRSKY

Elastic constants and other data for tall slender structures (*e.g.*, church towers) were obtained from study of displacements and strains. Three types of characteristic frequencies were investigated: rotation of the built-in ends; bending as a cantilever; and combined end rotation and cantilever bending.

A brief abstract of this paper, by H. W. Koch, is published in *VDI Zeitschrift*, V. 98, No. 7, Mar. 1, 1956, p. 276.

A philosophy on loading tests

D. B. DOREY and W. R. SCHRIEVER, *ASTM Bulletin*, No. 214, May 1956, p. 37-44

Includes a review of present building code requirements which indicates a lack of uniformity in loading tests specifications. Separates loading tests into three classes: acceptance tests, rating tests, and research tests; and defines the scope and purpose for each. Proposes a general method for the formulation of test loadings based upon such factors as type of structure, intended use of structure, expected type of failure, and effect of time and repetition

of loads in normal use. Concludes with criteria for assessing structural adequacy based on both strength and stiffness. Includes an extensive and up-to-date bibliography on the subject of testing.

Experiments on concrete under combined bending and torsion

NORMAN S. J. GRASSAM, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 2, Mar. 1956, pp. 159-165
Reviewed by ARON L. MIRSKY

On basis of a small number of tests, the maximum principal tensile stress theory, modified to allow for some plasticity, was found to approximate closely to observed values of fracture of concrete under combined bending and torsion. Interaction formula suggested is

$$(M/M') = 1 - (T/T')^2$$

where M' and T' are bending moment and torque, calculated on semiplastic basis (mean of fully elastic and fully plastic values), causing rupture when applied separately. M and T are bending moment and torque which cause fracture when applied together.

General

How snow affects roof structures (in Serbian)

MIODRAG MILOSAVJEVIC, *Naše Gradevinarstvo* (Belgrade), No. 1, 1956, pp. 31-33
Reviewed by J. J. POLIVKA

Refers to failures of or serious damage to buildings in Żelezare in 1954 caused by unusual depth of snow and presents suggestions on how to consider this type of live load for reliable safety. In many European countries the snow load is specified at 15 to 20 psf (75-100 kg per sq m) up to elevation 1000 ft. In accordance with the Yugoslav building code the snow load is to be increased with the increasing altitude, *i.e.*, S(in kg per sq m) $= 75 + (A - 500)/4$, from which the roofs in Żelezare at elevation 657 m(2150 ft) should be designed for 115 kg per sq m(23.5 psf) snow load. In that particular disaster area the maximum snow load was found to be 325 kg per sq m(66 psf) or, with a depth of 109 cm(3.53 ft), 300 kg per cu m(18.8 lb per cu ft). The failures occurred with relatively light steel and timber roof structures and it is evident that a live load increase of 46 psf could not cause failure of a concrete roof. Appropriate revision of the building code is recommended.

Failure and repair of Ridham Dock

ROBERT G. T. LANE and GORIUNE T. GREGORIAN, *Proceedings*, Institution of Civil Engineers (London), Part II, V. 5, No. 1, Feb. 1956, pp. 26-51 (including discussion)
Reviewed by ARON L. MIRSKY

Concrete gravity walls of dock, built in 1913, had undergone partial failure and repair in 1921 and 1931. Tidal surge of Jan. 31-Feb. 1, 1953 caused shift and tilt in west wall. Paper discusses causes and repair. (See also *The Engineer*, V. 199, No. 5184, June 3, 1955, pp. 772-773; "Current Reviews," ACI JOURNAL, Dec. 1955, p. 496).

No-joint concrete pipe poured continuously in trench

W. B. LENHART, *Rock Products*, V. 59, No. 5, May 1956, pp. 170, 171, 194, 196

A special machine forms pipe at the rate of 500 linear ft per 3-hr placing, using ready-mixed concrete delivered to the job. The machine consists of a winch-drawn horizontally sliding form which moves forward in the trench as concrete is dumped in a top hopper and vibrated into place around the form. The approximate cost of the equipment is given as $9000 for apparatus of 30-in. pipe diameter in size.

Growth of the concrete industry

A. L. L. BAKER, *The Engineer* (London), Centenary Number, Jan. 4, 1956, pp. 179-184
Reviewed by ARON L. MIRSKY

A brief but engrossing history of cement and concrete. Author examines particularly some of the factors which have influenced the development of cement and concrete, and especialily the reasons why the use of reinforced concrete and pertinent design theory has lagged in England (America, too) as compared to the European continent.

One sentence is eminently quotable: "The good craftsman loves his material,

and so must the concrete designer and all his associates, for on them lies a great deal of the responsibility of ensuring that the 100 million tons or so of cement now used annually do not deface the earth with a material which it is difficult to erase."

Are there any reasons to give preference to steel structures in relation to reinforced and prestressed concrete structures? (in Serbian)

BRANKO ZEŽELJ, *Naše Gradevinarstvo* (Belgrade), No. 1, 1956, pp. 29-30
Reviewed by J. J. POLIVKA

Refers to the same subject published in *Stroiteljnaja gazeta* (Moscow), and expresses the opinion of Yugoslav engineers. It is emphasized that the enormous progress in concrete in the past 20 to 30 years, not only as a material but also in design and construction methods, substantiates the preference in many cases. The following advantages of concrete are discussed: greater safety factor; practically no maintenance in comparison with steady and costly protection of steel against corrosion; economy of construction (with exception of some long-span structures) with high-strength, lightweight, and prestressed prefabricated elements; short erection time with standardized precast structural members; and better esthetic and artistic characteristics.

Steam hardening of concrete units (Feststellungen zur Dampfhaertung des Betons)

OTTO GRAF, *Betonstein-Zeitung* (Wiesbaden), V. 21, No. 9, Sept. 1955
Reviewed by WERNER H. GUMPERTZ

Experiments by Professor Graf tend to show that the use of saturated steam under atmospheric pressure in curing concrete block is particularly beneficial for porous and low-cement mixtures. Experiments indicate that steam curing with pressures of 130 psi is even more effective than atmospheric steam. The latter is especially so for concrete made of relatively fine aggregate. Professor Graf shows that various types of cement have substantially different effects on the concrete cured by high-pressure steam. He therefore recommends extensive testing of all precast concrete to determine the effect of steam curing.

Perforated concrete pipe airport subdrain

Concrete Pipe News, V. 8, No. 6, June 1956, pp. 66-68

Describes the use of perforated concrete pipe to drain the Richmond, Va. city airport. Includes specifications for the perforated pipe and its installation.

Notes on waterproofing buildings

ELYWYN E. SEELYE, *Consulting Engineer*, V. 7, No. 2, Feb. 1956, pp. 62-65
Reviewed by ARON L. MIRSKY

Descriptions and discussions of various methods of waterproofing, including warnings of lurking troubles. Practicing designers will want to clip and file this.

Fastening rails to concrete sleepers

Engineering (London), V. 181, No. 4703, Apr. 27, 1956, p. 280
Reviewed by ARON L. MIRSKY

Description of experimental steel clips which incorporate shock-absorbing synthetic-rubber pads to reduce wear. Fastening to concrete ties is by screws run into corrugated wood plugs cast in the tie, as developed in Germany.

North of Scotland hydroelectric schemes: The Glen Shira scheme

The Engineer (London), V. 201, No. 5230-5231, 1956: Apr. 20, pp. 364-368; Apr. 27, pp. 396-399
Reviewed by ARON L. MIRSKY

Another in series of articles* describing the North of Scotland hydroelectric projects. Glen Shira scheme is a peak-load development designed to yield 80,000,000 kwh using pumped storage. Dam is roundhead buttress type; Clachan power station, of reinforced concrete, was constructed in the rock of a steep hillside by cut-and-cover methods.

*See "Current Reviews," ACI JOURNAL, Mar. 1956, *Proc.* V. 52, p. 808

Control and inspection of ready-mixed concrete

E. L. Howard, *Rock Products*, V. 59, No. 6, June 1956, pp. 229-232

Discusses the function of control within the organization of a ready-mixed concrete plant. Particularly interesting data show that by the reduction of the coefficient of variation the producer is enabled to reduce cement content more than 1 sack per cu yd for 3000-psi concrete. Of particular interest is a table showing the required water-cement ratios to produce various compressive strengths with different brands of cement (with West Coast materials). The table indicates that new ACI Building Code requirements are quite conservative.

Cracking of concrete face brick and the development of data necessary for the establishment of criteria for its manufacture and installation

Building Research Advisory Board, Division of Engineering and Industrial Research, National Academy of Sciences—National Research Council, Washington, D.C., May 1956, 24 pp., $1.50

Reviews the investigations made by the committee and presents their recommendations. The investigations cover physical properties of the concrete masonry units; review of manufacturing methods and data; a review of existing standards and test procedures; and a study of construction methods and in-place performance in different geographic locations. The committee found that shrinkage cracking of face brick is not a problem in all areas of this country. Where such cracking is a problem, they recommend: (1) an improved quality brick; (2) placing in an air dry condition; (3) approved mortars; (4) the use of control joints and/or horizontal joint reinforcing; (5) additional research to establish adequate tensile strength criteria, properties of mortars, magnitude of shrinkage stresses, and a relation of the humidity conditions to the permissible moisture condition of the brick. In addition, the committee found a need for additional research on the same problems occurring with other concrete masonry units. The requirements for control joint spacings and amount of joint reinforcement are related and a sliding scale of suggested joint spacings, based on the amount of joint reinforcement used and the quality of brick employed, is suggested.

Sampling and testing ready-mixed concrete

Bulletin No. 66, National Ready Mixed Concrete Association, Washington, D.C., May 1956, 48 pp.

This publication replaces two earlier NRMCA publications on the same subjects. It discusses the requirements for sampling and testing, as specified in ASTM standards, with various precautions to insure that the tests for acceptance of ready-mixed concrete be fair to the ready-mix supplier.

Leakproofing inland waterways, basins, and supply trenches (Etanchement des canaux de navigation interieure, des etangs et des rigoles d'alimentation)

L. Geoffroy, *Annales des Ponts et Chaussees* (Paris), V. 126, No. 2, Mar.-Apr. 1956, pp. 189-206

Reviewed by Aron L. Mirsky

Describes and compares various methods of reducing leakage: concrete screens (cutoff walls), grouting, bituminous coating, sheet piles, asphalt mastic, and reinforced concrete lining.

How to tool concrete

Constructional Review (Sydney), V. 29, No. 4, Apr. 1956, pp. 35-37, 42

Describes the operations of hand or power tooling as a decorative concrete finish to obtain textured surfaces. Bush hammering and point tooling are described and the results illustrated.

Polyvinyl acetate cement compositions

William D. Robinson (E. I. du Pont de Nemours & Co.), U. S. Patent 2,733,995, Feb. 7, 1956

Ceramic Abstracts
June 1956 (Barbour)

In the use of polyvinyl acetate spray-dried emulsion with portland cement in

water-dispersible paints for masonry surfaces, the polyvinyl acetate obviates the necessity of wetting the wall before application and promotes better adhesion. The patent includes the ratio of 1 part of dry polyvinyl acetate to 1 to 10 parts of portland cement by weight, the preferred range being 4 to 6. The cement portion can also include pigments, modifiers, or waterproofers.

No resale value in second hand concrete

E. L. Howard and James Ida, *Rock Products*, V. 59, No. 8, Aug. 1956, pp. 277-280

Discusses the value of quality control for concrete as established by the service department of ready-mixed concrete operators. Includes some interesting notes on test results of companion specimens with the method of delivery to the laboratory as a variable. Discusses the value of the impact rebound hammer in this work.

History of concrete

Concrete and Constructional Engineering (London), V. 51, No. 1, Jan. 1956, 262 pp.

This 50th anniversary issue takes the form of a symposium by some of the leading authorities on concrete, who describe progress of the material in their various countries. Apart from descriptions of both old and new structures and techniques, the various authors offer many interesting and worthwhile comments.

Is it justified to give preference to concrete over steel in structural design? (in Serbian)

M. Rodojkovic, *Naše Gradevinarstvo* (Belgrade), V. 10, No. 5, May 1956, pp. 669-671

Reviewed by J. J. Polivka

Discusses this question considering present conditions in Yugoslavia. Today there is sufficient production of steel there so that the most economical material can be chosen in any structural design. Serious disadvantage of steel structure is the relatively smaller safety factor, especially in cases where live load (*e.g.*, snow) can exceed the assumed intensity. Each case should be thoroughly investigated under all conditions.

Hydroelectric management of the Grande Dixence (L'amenagement hydro-electrique del al Grande Dixence)

Eric Choisy, *Memoires*, Societe des Ingenieurs Civils de France (Paris), V. 109, No. 2, Mar.-Apr. 1956, pp. 113-123

Reviewed by Aron L. Mirsky

General description of the project from conception to construction.

Placing concrete in a deep mine

B. A. Lamberton, *Civil Engineering*, V. 26, No. 8, Aug. 1956, pp. 37-39

Describes the use of prepacked concrete for lining shafts of a gold mine 3000 ft below the surface in South Africa. The grout developed on the job consisted of approximately a 1 to 2 mix of pozzolanic cement and blended sand, plus 1 percent admixtures, giving a final water-cement ratio of about 0.5.

Redriving characteristics of piles

Nai-Chen Yang, *Proceedings*, ASCE, V. 82, SM3, July 1956, pp. 1026-1 to 1026-19

Discusses the effects of the temporary stress adjustments in the subsoil after pile driving which, to some extent, invalidate the application of pile-driving formulas. Presents a formula for predicting the load capacity of a pile based upon observations of redriving. Concludes that such redriving observations may be used as a relatively inexpensive substitute for pile loading tests.

Concrete mix design for vibration

Highways and Bridges and Engineering Works (England), V. 23, No. 1131, Mar. 21, 1956, pp. 6, 8

Highway Research Abstracts May 1956

Many mixes which behave admirably under mild treatment in hand compaction show signs of severe segregation or violent rotational instability when vibrated. Tests in progress are designed to determine the resistance of various gradings to segregation and rotation, not their desirability from other aspects such as workability.